In and Out of Hungary: Paul Erdős, His Friends, and Times

by

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Abstract

This is a biographical sketch of Paul Erdős, with an emphasis on the historical background which affected the extraordinary geographic and mathematical trail of the most prolific mathematician of our time.

The first part is an essay on some aspects of the career of Paul Erdős. We focus on Erdős's formative years and influences, and on his personal and professional ties. The second part is a year-by-year account of random events and anecdotes, based largely on two interviews with Erdős. The material is supplemented with lists of honors received by Erdős, as well as a list of his coauthors with the most joint papers.

While we do make occasional references to mathematical disciplines, we have intended this article to be accessible to non-mathematicians.

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Property is nuisance.
Paul Erdős

Paul Erdős is the most prolific mathematician of our time. Except for Leonhard Euler (1707-1783) whose *Oeuvre* stretches over 90 large volumes, no great mathematician in history matches Erdős's list of over 1400 papers, several of which gave rise to new branches of mathematics.

Other "mosts" that the reader may encounter in any essay about Erdős: he traveled the most miles (indeed, he is constantly on the move across and between continents) and he collaborated with the most mathematical partners (more than 250 of them). The "leasts" are equally extraordinary for a scientist of his stature: according to the legend, he owns barely more than a change of clothes, he has no home, no bank account, no year-round source of income[1]. Taken literally, all these statements are false, yet Erdős

[1] For myths like these, see the subtitle of a 1984 article in *Science* [38].

would indeed have as little to do with his modest property as possible. He travels the world with a half-empty suitcase. He owns a condominium apartment in Budapest and has even willed it to an adopted nephew. But Erdős himself has hardly used the apartment in the past two decades; he would let friends and visiting colleagues use it free of charge. Erdős has at least two bank accounts, one in Budapest and one in New Jersey, but friends do most transactions for him, they even sign his checks. Erdős has been affiliated with the Mathematical Institute of the Hungarian Academy of Science since 1962 but the pay associated with that job never worried him[2].

Most of what has been written about the ways of Paul Erdős, in the U. S. and in the world, shows him to be a thoroughly international thinker, his mind apparently free of national boundaries. Comparatively little attention seems to have been given to his relationship with Hungary, his native country. However, a sketch of this complex link is requisite to a portrait of Erdős.

News-writers tend to sensationalize Erdős's eccentricities and child-like dependence and describe him as the single-minded champion of an arcane world (mathematics, that is), totally consumed by his passion for a "narrow" pursuit.

His mathematical friends around the world know better. They accept his innocence and care for him fondly, repaying in small ways for the warmth and the light he brings into their offices and homes. They also know that, far from being a mathematical robot, Erdős has always been mindful of his surroundings, small and large.

Erdős's compassion for his fellow mathematicians is as legendary as the trail of joint publications marking the path of his Brownian motion. A man without earthly possessions, he always finds a way to help colleagues in need, even total strangers. Of his $50,000 Wolf prize (1984), by far the largest cash award he ever earned, he kept only $720. He used $30,000 to endow a postdoctoral fellowship at the Technion (Haifa) in the name of his mother, and gave the rest to needy relatives and colleagues. On the occasion of Erdős's 70th birthday, Ernst Straus observed [36]:

> "No appreciation of Paul Erdős would be complete without remembering his great generosity and care for his fellow man, as well as his total devotion to the rights of human beings over

[2]For more than two decades, most of Erdős's time was recorded as unpaid leave of absence.

all other loyalties. His money and connections are available to promising students and mathematicians at all levels. He has made some extraordinary personal sacrifices rather than to acquiesce in even relatively minor violations of rights."

Conjecture and proof

$$cn \log\log n$$

Paul Erdős

The presence of Paul Erdős at formal or informal mathematical gatherings always stirs excitement. Wherever he goes, his lectures draw crowds. His following has grown exponentially through the years.

Then there are his mathematical foes, those who dislike his mathematical style. Saunders Mac Lane[3] made these ponderous comments on the pages of the *Mathematical Intelligencer* [30]:

> "...the difficulty may lie in emphasizing too much of a Hungarian view of mathematics – that the science consists not in good answers but in hard questions. This emphasis on problems is likely to lose track of the fact that what matters most about a problem is its relevance."

"Who brought this disgrace upon us?", fumed Erdős's Hungarian adversaries (oh yes, they do exist), fluttering copies of Mac Lane's tract and lamenting the tarnished honor of a nation. Who, indeed, other than Paul Erdős, the virtually inexhaustible fountain of simply-stated hard problems in number theory, geometry, complex analysis, constructive functions, set theory, topology, graph theory, combinatorics, and more? Paul Erdős, who takes such a delight in his questions that he will not pause to tell, or ask, why?

[3] In 1985, Mac Lane conducted a seminar at the University of Chicago under the title "Good mathematics and bad mathematics – the latter illustrated by examples from combinatorics." He invited Lovász and this writer to present an opposing point of view, which we did rather successfully. It must be a mere coincidence that both of Mac Lane's appointed opponents happened to be countrymen of Erdős. – Mac Lane refers to Erdős as his "old friend." On a visit to the University of Chicago in 1986, the first request of Erdős upon arrival was to be shown to Mac Lane's office, where he immediately began talking about mathematics even while changing his socks to soothe his sensitive skin.

Erdős has never declared a program of research nor stated a general mathematical objective. Presumably he would have difficulty putting together an acceptable grant proposal. Straus writes [36]:

> "In this century, in which mathematics is so strongly dominated by 'theory constructors', he has remained the prince of problem solvers and the absolute monarch of problem posers. ... [A] great mathematician ... complained to me that 'Erdős only gives us corollaries of the great metatheorems which remain unformulated in the back of his mind.' I think there is much truth in that observation, but I don't agree that it would have been either feasible or desirable for Erdős to stop producing his corollaries and concentrate on the formulation of his metatheorems. In many ways, Paul Erdős is the Euler of our times. Just as the 'special' problems Euler solved pointed the way to analytic and algebraic number theory, topology, combinatorics, function spaces, etc.; so the methods and results of Erdős's work already let us see the outlines of great new disciplines, such as combinatorial and probabilistic number theory, combinatorial geometry, probabilistic and transfinite combinatorics and graph theory as well as many more yet to arise from his ideas."

Indeed, while some of these theories, such as *extremal graph theory* (cf. [3]), gradually unfolded over the decades through Erdős's thousand results and tens of thousands of questions, others emerged as full-blown theories in monumental papers such as the sixty-page paper by Erdős and Rado on *partition calculus* (Ramsey theory, especially in uncountably infinite cardinals and ordinals) (*Bulletin of the American Mathematical Society,* 1956) (cf. [17]). Erdős has also been the universal thesis advisor: it is believed that at least a hundred mathematicians have reached tenured positions by working on problems he has proposed.

Since his college years, Erdős has been a catalyst of interaction between mathematicians, making introductions wherever he sensed shared mathematical interests. (This writer has been the beneficiary of several such introductions.) In his commemorative address for mathematician victims of the Holocaust [42], Turán mentions two mathematicians (S. Sidon and P. Csillag) both of whom he had met through Erdős, three years his junior. Turán's introduction to Sidon, author of classic work in the theory of trigonometric series, has become a legend.

A reclusive man with a severe psychological handicap, Sidon was employed by Generali Insurance Co. until his inexplicably early retirement in 1934. Erdős met him in 1932 through fellow student Sándor Kemény who worked for Generali. Impressed by Sidon's questions, Erdős felt compelled to take Turán to Sidon's home. "... as usual, [our meeting was] initiated and arranged by Erdős," Turán writes. When the two students showed up at his doorstep, Sidon greeted them with the words, "Please visit another time and *especially* another person ..."[4] Eventually Erdős prevailed, they managed to talk to Sidon and came back several times[5].

Erdős was captivated by one of Sidon's inventions, and a theory of "Sidon sets," typical of Erdős-style additive number theory, began to take shape. A problem on Sidon sets became the subject of one of the early joint papers of Erdős and Turán (1941).

At his first meeting with Erdős, Sidon proposed a problem on the upper density of certain integer sequences. Twenty years old, Erdős boasted that he could probably solve it. He was right, but it took him another 20 years to do so (*Acta Scientiarum Mathematicarum, Szeged,* 1954). His solution demonstrated the *existence* of certain sets of integers by showing that "almost every set" had the desired property; no explicit construction for such a set has yet been found. This was not the first time that Erdős used the *probabilistic method*. However, Turán observes that, from that point on, the method became more prevalent in Erdős's work [39, p. 9].

Through collaboration of unprecedented dimensions in the history of mathematics, Erdős continues to shape the mathematics done around the globe. An indefatigable prophet of the jet-age, he visits his ever growing circle of disciples, coast to coast, continent to continent, with unpredictable regularity. "Want to meet Erdős? Just stay where you are and wait. He'll show up," the common advice goes.

There is no way to do justice to even a fraction of the more than 250 coauthors and countless friends of Erdős in such a short account. Those omitted include a number of his major collaborators and close friends[6]. I

[4] Erdős likes to point out that the phrase has more punch in its original Hungarian: "Kérem, jöjjenek máskor és különösen máshoz." The last four words correspond to the last six in English.

[5] The recollection of Erdős of the date is at variance with Turán's account. Erdős puts this egregious encounter at 1937, several years after he had introduced Turán to Sidon. A variant of this anecdote, appearing in [2, pp. 200-201], seems to be closer to Erdős's version.

[6] The more glaring omissions include Erdős's intimate friendship with two of his outstanding collaborators of the past three decades, Miklós Simonovits and Endre Szemerédi.

regret this imbalance which I believe was inevitable, given the limited scope of this writing. The reader will notice a bias towards Hungarians which is partly due to this writer's observation point. It may be somewhat surprising, however, that such a bias is indeed quite prominent in the career of Erdős. In spite of the fact that, after age 23, Erdős spent less than 10% of his time in Hungary and the great majority of his coauthors are from other parts of the world, encompassing over 25 countries, several of his Hungarian collaborations stand out. His influence on the mathematical profile of his native country is immeasurable.

In and out of Hungary: menacing worlds

> *Children become letters.*
>
> Anyuka (Anna Wilhelm Erdős, the mother of Paul Erdős)

For much of his peripatetic career, politics of the powers of the world had a considerable effect on Erdős's trail. Facing government authorities ranging from the merely annoying to the truly menacing, Erdős never allowed them to hold the upper hand, to constrain or humiliate him.

As a Jew growing up in the increasingly hostile inter-war social climate, he knew, by age 12, that eventually he would have to leave Hungary.

That moment arrived during his visit home in the summer of 1938, at the age of 25. Erdős had held a fellowship in Manchester from 1934 to '38 and visited home three times each year. No longer able to travel through Austria after the March 1938 *Anschluss*, he returned from Britain to Hungary the long way around through Italy for his 1938 summer visit. He was planning to travel to the U. S. at the end of September to take up his fellowship at the Institute for Advanced Study in Princeton.

Back home, he monitored local and foreign news broadcasts. The morning news of September 3 was particularly ominous, indicating that Nazi Germany's appetite now included the *Sudetenland,* part of Czechoslovakia. Alarmed by the possibility of an imminent dramatic turn of events, Erdős decided to leave immediately. Several weeks ahead of schedule, he packed in a great hurry, bade his parents farewell at the railway station, and crossed the border the same day. That was the last time he saw *Apuka*[7], his beloved father.

[7] *Apuka:* Daddy. Pronounced "oh-poo-koh."

He traveled to Britain and spent three weeks there, visiting Cambridge, Manchester, and London, among other places. While in Reading on September 12, he heard Hitler's inflammatory speech on the radio. No longer in danger of being trapped in Hungary under the shadow of the Third Reich, Erdős hoped for a rapid outbreak of war, and for Hitler's quick demise. He was planning to offer his services to Britain's war effort.

But war did not break out. Prime Minister Chamberlain elected to appease Hitler, betraying Czechoslovakia. Erdős was on his way to Princeton[8] aboard the liner "Queen Mary" when the news of the conclusion of the Munich pact reached him on September 30.

It wasn't until a decade later that Erdős returned to Budapest, mourning his many friends and relatives who had fallen victim to the Holocaust. He rejoiced at seeing the survivors whose number included *Anyuka*[9], his beloved mother as well as his cherished friends Tibor Gallai and Paul Turán.

On this visit, Gallai introduced a young star to Erdős, then undergraduate Vera Sós, formerly Gallai's student in high school, and before long, wife of lucky Turán. After many years of close friendship and occasional collaboration, Sós became one of Erdős's major collaborators in the late 70s.

Back in 1948, the Soviet-orchestrated conversion of Hungary to a Communist state was in full swing. By the time the borders were hermetically sealed and the scripted trials began in 1949, Erdős had returned to the U. S. He did not return to Soviet-dominated Hungary until 1955, two years after Stalin's death, when his friends, especially George Alexits, a Fourier analyst with high connections, managed to arrange a special deal for him with the Hungarian government. Erdős, ever without a permanent domicile, was thereafter listed in his passport as a Hungarian citizen and Israeli resident. The government's promise to let Erdős move freely was reiterated under the Kádár regime which had been installed by the Soviets after they crushed the revolution of 1956. In the early 60s the special status of Erdős was formalized in a "consular passport," allowing him to enter and leave Hungary at any time. This was a singular privilege not only in 1955 but for the subsequent three decades.

The guarantees were meager, though. The passport could be revoked on the whim of the communist leadership, should they choose to ignore

[8] A comment by Erdős on what he found in the New World: in 1938, some hotels in the U. S. did not permit Jews.

[9] *Anyuka:* Mommy. Pronounced "on-yoo-koh."

the predictable protest of Western colleagues. Apparently, however, Erdős's assessment of the situation was adequate, "Joe"[10] never backed out of the deal.

From then on, Erdős returned to Hungary with increasing frequency, and these visits allowed him to initiate and pursue some of his most productive collaborations with his characteristic vigor.

The next decade saw the emergence of about two dozen joint papers on a variety of subjects with number theorist, analyst and probabilist Alfréd Rényi; these include their seminal study of the "Evolution of random graphs." With his old friend, number theorist and analyst Paul Turán, Erdős started their famous series on "Statistical group theory" while the two also collaborated on a number of other subjects, including interpolation and diophantine approximation. "Partition relations for cardinal numbers" (1965) was a milestone in transfinite combinatorics, written with set theorists András Hajnal and Richard Rado. Referred to by colleagues as the "Giant Triple Paper," this was but one among a dozen papers with Hajnal in this period.

Turán introduced his student András Sárközy to Erdős around 1963; Sárközy brought his colleague Endre Szemerédi along. Between 1966 and 1970, a dozen Erdős-Sárközy-Szemerédi papers appeared, marking a new era in combinatorial number theory. To date, Hajnal and Sárközy each have written over 50 papers with Erdős, yet there is not even a remote connection between the subject areas of the Hajnal papers and those of the Sárközy papers.

Feasting on problems of Erdős, Szemerédi has emerged as one of the most formidable problem solvers of our time. His crowning achievement was his proof, in "a masterpiece of combinatorial reasoning" [21, p. 46] of the 1936 conjecture of Erdős and Turán that sets of integers of positive upper density contain arbitrarily long arithmetic progressions (*Acta Arithmetica* 27 (1975), pp. 199–245). Two of Erdős's closest associates, Hajnal and Ronald Graham, were so enthusiastic about the result, they both liberally donated their time helping Szemerédi write up his most celebrated paper[11]. Szemerédi cashed the thousand dollar check Erdős had offered for the solution (the biggest

[10] "Joe" is the nickname of the communist regimes in Erdőshese, referring to Joseph Stalin.

[11] The first draft was written by Hajnal; Graham produced the final version. In his paper's *Acknowledgment*, Szemerédi expresses "extremely great indebtedness" to Hajnal and Graham, because "they wrote the whole paper." Remarkably, both Hajnal and Graham, tireless leaders of mathematical communities and dedicated administrators, volunteered their time for this endeavor amidst myriad other obligations.

monetary reward[12] Erdős ever paid).

Szemerédi's work has been influential not only in combinatorics and number theory, but also in the *theory of computing*, an area in which Erdős claimed little interest. Ironically, developments of the past two decades in this field have put the emphasis on asymptotic results, often with a heavy combinatorial core, a combination that has made Erdős's sublime universe eminently relevant.

Child prodigies

I am the epsilon *from Szeged.*

14-year-old math whiz Attila Máté introducing himself to *Anyuka* over the phone

Erdős has always been eager to meet mathematically gifted children. With her long tradition of talent search, Hungary has been particularly well suited to meet this desire of Erdős. And Erdős would never miss a chance to let new *epsilon*s – Erdőshese for young talent – try their claws on his problems. Once a child prodigy himself, he builds very special relationships with those who have shown their gift early. He nurtures their talents and introduces them to open problems they could handle. Erdős talks fondly about their results [10] and continues to monitor their careers long after their period of frequent correspondence has ended.

They are his children.

His favorite adoptee is *Lajos Pósa*. Pósa was 12 when logician Rózsa Péter introduced him to Erdős in 1959. Over lunch Erdős challenged Pósa to *prove* that among any $n+1$ integers between 1 and $2n$, there are two that are relatively prime[13]. After brief contemplation, with his spoon stopped in mid-air, little Pósa uttered the elegant proof, consisting of a single short

[12]Since the early 50s Erdős has been offering monetary awards, ranging from \$10 to \$25,000, for solutions to his problems. The biggest awards claimed so far have been Szemerédi's \$1,000 and four awards of \$500 each. The recipients of the \$500 awards were Hugh L. Montgomery and R. C. Vaughan (number theory), Péter Frankl and Vojtech Rödl (combinatorics), Helmut Maier (number theory), and very recently, Rudolf Ahlswede and Levon Khachatrian (combinatorics).

[13]Characteristically for Erdős, this innocuous question is a special case of a problem which has still not been fully resolved: given k, determine the maximum number of integers between 1 and n such that no k are pairwise relatively prime. The question to Pósa gives the tight solution for $k = 2$. Substantial progress on the general case was made recently by R. Ahlswede and L. H. Khachatrian, *Acta Arithmetica* 66 (1994), 89-99.

sentence[14] [10]. Erdős was impressed. He continued to pose questions to Pósa by mail and was overjoyed by Pósa's interest in his questions. Gradually he introduced him to graph theory. In 8th grade, Pósa already produced *conjectures and proofs* of his own. Big trauma hit Pósa around that time; his mother died of cancer. This, perhaps, made his intense involvement with Erdős even more significant.

Pósa's first paper on extremal graph theory appeared in Hungarian in 1962. His influential second paper, on Hamilton paths, appeared in English in 1962 (cf. [3]). Erdős helped the 14-year-old author to turn the results into papers. Their first joint paper, on disjoint cycles in a graph, also appeared in 1962. A number of unpublished results of Pósa from his early high school years appear in Lovász's problem book [29].

Pósa entered Fazekas High School of Budapest in 1961 as a 9th grader, a member of the inaugural year of a special program for mathematically gifted students at that high school. A teacher by instinct, Pósa immediately began to propagate Erdős's world of thoughts among his classmates whose number included an astonishing array of now well known mathematicians, László Lovász, Katalin Vesztergombi, József Pelikán, Miklós Laczkovich, Zsolt Baranyai (1948-1978), István Berkes, Péter Major, and Hajnal Andréka, among them.

This unique gathering place of young talent required a special curriculum. The mathematics curriculum was developed by a powerhouse of volunteers that included some of the closest associates of Erdős and clearly bore Erdős's imprint. One of the novel elements of the curriculum was combinatorics and graph theory. The textbook, probably the first high school text in the area, was written by András Hajnal[15] in 1965. Graph theorist and legendary teacher Tibor Gallai[16] reviewed and commented on each chapter, ensuring its accessibility to the students. Erdős provided the exercises and Hajnal himself taught the class in 65-66; the students also had a chance to meet Erdős and Gallai.

László Lovász did not start as early as Pósa, his classmate. Lovász wrote his first significant papers "at the ripe old age of nearly 17," Erdős comments [10]. By the end of high school, Lovász, Pelikán and Vesztergombi coauthored a lovely text on combinatorics for *elementary school* stu-

[14] "Two are neighbors."

[15] See the section on "The making of disciples" for the story of the Erdős–Hajnal collaboration.

[16] If there is a single person Erdős would call his closest friend, it is Gallai (1912–1992). Cf. the section on "Anonymous" as well as Vera Sós's story in "The making of disciples."

dents (up to 8th grade). Gallai became one of Lovász's mentors. Lovász's mathematical style owes much to Gallai. Lovász, one of the most influential combinatorists of our time, has done a great deal to make Gallai's work (including some of his unpublished results) widely known and appreciated.

It was at the home of *Anyuka* where Pósa first met another child prodigy, one year his senior. *Attila Máté,* "the *epsilon* from Szeged[17]," was 13 in 1959 when Erdős sought him out. Erdős and Hajnal traveled to Szeged to work with set theorist Géza Fodor when they heard about the remarkable 7th grader. Erdős wanted to meet him immediately, so they got word to his school that they would like Máté to walk over to the university. Little Máté was no less eager to hear new problems than Pósa [10], so Erdős kept him busy in correspondence. The problems were mainly in graph theory and number theory. Máté visited Budapest several times a year to attend the "Young Mathematicians' Club" meetings at the headquarters of the Bolyai Society; on such occasions he would call Erdős and visit if he was around. On one occasion in 1960, Erdős invited little Máté for lunch at a fancy hotel in Budapest. Máté met a number of mathematicians there, including Paul Turán and Vera Sós.

Meanwhile Fodor introduced Máté to set theory. He was in the 10th grade and no longer clueless when he attended Hajnal's D. Sc. thesis defense (an act somewhat analogous to the German *Habilitation*) where Hajnal presented his proof of a set-theoretic conjecture of Ruziewicz. The problem concerned independent sets for set mappings over sets of singular cardinality. The problem had previously been solved by Erdős under GCH[18], an assumption Hajnal's proof did not require.

Máté graduated from high school in 10th grade, two years ahead of normal schedule. At 16, he enrolled at the University of Szeged. By then, he was doing graduate level work. He worked on set theory with Fodor in Szeged and later in Budapest with Hajnal. Although he has few joint papers with Erdős, several of his papers relate to the work of Erdős both in finite and in transfinite combinatorics. At the International Congress of Mathematicians[19] in Vancouver in 1974, Erdős, Hajnal and Rado approached him with the suggestion that he join their team in writing a monograph on *partition calculus*. Máté agreed and subsequently became the engine behind the project, putting in several years of work. The book

[17]Szeged: university town in Hungary, home to Frederick Riesz and Alfréd Haar.
[18]GCH: the Generalized Continuum Hypothesis.
[19]Major quadrennial gathering of mathematicians.

eventually appeared in 1984 [17].

Meanwhile, in an unusual major shift, Máté abandoned set theory and turned, very successfully, to classical analysis where he continues to thrive to this date. His entry into the field was marked by a joint paper with Névai which appeared in the *Annals of Mathematics* and is related to a 1940 paper in the same journal by Erdős and Turán, "On interpolation III." This may be a coincidence, though; Máté never discussed analysis with Erdős.

Another child prodigy, 15-year-old *Imre Ruzsa,* was presented to Erdős by Pósa, his mentor, in 1968. Within a year, a joint paper by Erdős, Sárközy and Ruzsa on additive functions was conceived. Ruzsa was enchanted by combinatorial number theory, and went on to enchant the world with his contributions to the field.

Béla Bollobás was 14 in spring, 1958 when he first attended a lecture by Paul Erdős at a session of the "Young mathematicians' club." A few months later, during another brief visit to Budapest, Erdős learned about the promising *epsilon* who by then had won all the math competitions of his age group. Erdős invited the ambitious boy to lunch at his hotel, Hotel Golf, talked with him about mathematics, and impressed him with his kindness. Of course *Annus néni* ("Aunt Anna," Erdős's mother) was also present. From then on, Bollobás visited them whenever Erdős was in Budapest. Bollobás was 17 when he and his parents first invited Erdős and *Annus néni* over to their home for lunch. This became a recurring event, and Bollobás's father, a physician, examined *Annus néni* on these occasions. Even when Erdős was away (which was most of the time), Bollobás occasionally visited *Annus néni* and her sister in their apartment in Abonyi street. He got a lot of attention, and was always treated to cake and hot chocolate.

Bollobás's first joint paper with Erdős arose from a problem of Erdős which he and Erdős solved independently. Bollobás was 18, the subject was extremal graph theory. He probably did not suspect that after a detour into distant areas of mathematics, extremal graph theory would become one of his main trades, and the definitive monograph on the subject would be his work 17 years later.

Through Erdős's old friend Harold Davenport, Fellow of Trinity College, Cambridge, Bollobás got a stipend to spend the third year of his undergraduate studies at Cambridge University. Back in Budapest, Bollobás graduated from Eötvös University in 1966, spent a year in Moscow working in functional analysis under I. M. Gel'fand, and returned to Cambridge in 1969. He received a prestigeous fellowship and did his Ph. D. at Trinity College on "Numerical ranges in Banach algebras" (1971). He stayed at Trinity and

after 20 years became a Life Fellow there. In 1972 Bollobás and Erdős determined the correct speed of the parameter in the fundamental Erdős–Stone theorem. "Our best paper. We wrote it when collaborating by mail," Bollobás comments. Their collaboration peaked in 1976 when for a few months Erdős was Bollobás's guest at Trinity. It was during these months that Bollobás's wife Gabriella, a sculptor, did Erdős's bust.

More than any of the other former *epsilons* mentioned in this section, Bollobás became a disciple of Erdős. His second major monograph is on *Random Graphs* (1985), another field of Erdős's creation. In 1992 he launched a journal entitled *Combinatorics, Probability, and Computing*, linking two of Erdős's fields of interest to an important recent application area of Erdős–type thinking.

Ever since Erdős's 65th birthday, Bollobás has been organizing an Erdős–meeting at Cambridge every five years, always making sure that the exact birthday, March 26, falls within the meeting.

In and out of Hungary: paranoid bureaucracies

Another roof, another proof.
Paul Erdős

In the era of Iron Curtain isolation, Erdős, with his extraordinary capacity to disseminate problems and results, single-handedly maintained a powerful channel of communication between a substantial part of the Hungarian mathematical community and its western counterparts.

Painful as it was for him to leave his many Hungarian friends and protégés, including *his* child prodigies, for a long period of time, he did it once again in 1973.

In that year, the Bolyai Society celebrated his 60th birthday with a large international conference on "Infinite and Finite Sets" in Keszthely, Hungary. The festive event was marred by the involuntary absence of Israeli mathematicians, including invited speakers, friends and long-time collaborators of Erdős.

Following Soviet dictate, Hungary had severed diplomatic ties with Israel after the 1967 Middle East war. In spite of petitions by members of the Bolyai Society and the Academy of Science, the Hungarian government denied entry visas to Israeli mathematicians. Privately, some officials would cite "overriding foreign policy interests" (i. e. Soviet sensitivities). Erdős

was not moved by the excuse. In protest, he stayed away from Hungary for several years after the meeting.

At a conference in Paris in July 1976, Vera Sós, by then a close collaborator of Erdős, tearfully reported to him on the dramatic deterioration of her husband's health. Erdős lifted his self-exile and returned to Budapest for two weeks in August 1976 to see Turán. They talked at length for one last time. The subjects, Erdős recalls, included prime numbers as well as the (posthumous) publication of the greatly revised and expanded English edition of Turán's famous 1953 monograph on applications of his power-sum method [43], a project Turán had been working on for two decades. Three weeks later, the dear friend passed away.

Just as in his dealing with "Joe," Erdős demonstrated his principles and resolve in his brush with "Sam"[20]. Following Michael Golomb's letter [20] and Erdős's own account, here is the story of the incident that deprived the U. S. mathematical community of the presence of Erdős for nine years, and Erdős of his last regular job and his permanent resident status in the U. S.

A member of the mathematics department of Notre Dame University at the time, Erdős wanted to attend the International Congress of Mathematicians held in Amsterdam in 1954. As an alien, Erdős needed a reentry permit that would allow him to return to the U. S. In what should have been a routine process, Erdős had to undergo lengthy questioning by an Immigration officer. To the government's credit, Erdős emphasizes their courtesy in sending the officer from Detroit to Notre Dame. The interview took place in the office of Erdős, the last office he ever called his own.

He learned that a file had been compiled on his correspondence with mathematicians outside the U. S., on his expressions of sympathy for various causes, etc. In particular, the U. S. government knew about his correspondence with a person in Communist China (renowned number theorist Lo Ken Hua). Erdős had to answer such questions as to whether he had read works by Marx, Engels, Lenin, Stalin, and what he had thought of Marx. Erdős said that the only thing he had read was the Communist Manifesto, and that although he was not competent to judge, he thought Marx clearly was a great man. He was also asked whether or not he would return to Hungary if he could ascertain that they would let him out again. Erdős's response: "Of course, my mother lives there and I have many friends there."

Apparently these answers were deemed to hold menace to the security of the land of the free. The reentry visa was denied to Erdős. Still on American

[20] "Sam": Erdőshese for the U. S.

soil, Erdős had the choice between the security of his job at Notre Dame and his freedom to travel. He chose the latter. In subsequent years, his requests for a visitor's visa to attend conferences in the U. S. were repeatedly turned down except for a "special visa" he received in 1958 to attend an A. M. S. meeting[21] in Colorado. He wrote to friends around 1962 that apparently "U. S. foreign policy is adamant on two points: non-admission of Red China to the U. N. and non-admission of Paul Erdős to the U. S."

In 1963, a friend wrote on his behalf to the Immigration Service, asking to reviews his case. The government replied that Erdős had joined "proscribed organizations." With the help of Harold Davenport, Erdős composed a response, the main point of which was that the only organizations he had joined were the American and the British Civil Liberties Unions. His case was subsequently reviewed. He had to clarify one remaining worrisome issue: his ties to the Hungarian Academy of Science. Having given an apparently satisfactory explanation for this grave question, Erdős finally was permitted to return to the States. On his return, he informed his audiences: "Sam finally admitted me because he thinks I am too old and decrepit now to overthrow him."

Meanwhile the "green card" (U. S. permanent residence permit), which Erdős had acquired in 1948, had expired. Erdős never applied for another, although this meant considerable inconvenience. To this date, as a Hungarian citizen, he has to apply for a visa for his trips to the States; the visa needs to be periodically renewed.

> "It is a sad commentary on our time and country that this man – so totally immersed in scholarly work, so remote from the political arena, a free spirit who lives by the highest moral standards – could be harassed by bureaucrats in high position whose duty is to protect our freedoms." *Michael Golomb* [20].

Linguistics and other random comments

G. was captured in Hungary.

Paul Erdős, of a brilliant young colleague who found more than just mathematical beauty during his one-year visit to the Mathematical Institute in Budapest.

[21] *A. M. S.:* American Mathematical Society.

His often mispronounced name is Pál (Paul) Erdős, or Erdős Pál in the Hungarian order of family name - given name. His close friends call him "E-P," and those under sixty alternate in saying "E-P" or "Pali bácsi" [pron. "paw-lee but-chee"], meaning Uncle Paul.

The word "erdős" means "wooded" and is not an uncommon name in Hungary. It is pronounced approximately like "air-dish" where the "i" in "dish" sounds like that in "first." Note the pair of long accents on the "ő," often (even in Erdős's own papers) by mistake or out of typographical necessity replaced by "ö," the more familiar German Umlaut which also exists in Hungarian. (We salute Donald E. Knuth for including the long Hungarian Umlaut among his international characters in the TEX typesetting program; its code is `\H{o}`. Knuth's TEXbook uses the name of Erdős to illustrate the use of the character, `Erd\H{o}s`.)

Erdős has done much to connect Hungarian mathematicians to the rest of the world. At the same time, he has helped make Hungary an attractive place to visit if you work in number theory, combinatorics, set theory, constructive function theory, and probability theory, among other fields. Some of the major American collaborators of Erdős feel as much at home at the Mathematical Institute in Budapest as at MIT or Bell Labs. Ron Graham, Fan Chung, Joel Spencer, Ralph Faudree, Tom Trotter, and many other "honorary Hungarians" are thoroughly integrated into the community of Erdős's Hungarian friends, down to the latest gossip. Spencer and Fred Galvin even learned Hungarian, no small feat, Hungarian being linguistically unrelated to the Indo-European languages. The name of Erdős has attracted hundreds of American youths to the *Budapest Semesters in Mathematics* program during the last decade; the network of enthusiastic "honorary Hungarians" was instrumental in successfully launching the program in 1985.

Speaking of linguistics, a comment on "Erdőshese," a dialect existing in several languages may be in order. "How is the *epsilon?*", Erdős would ask a colleague, referring to a baby or a little child. "Who are the new *epsilons?*", he would ask his friends, inquiring about emerging mathematical talent (and he would immediately invite those named, often high school students, for a mathematical conversation). "Who is preaching?", he inquires about the name of the seminar speaker. "It's a pity Pósa died at such a young age," he laments the fact that at the age of 17, Lajos Pósa, his favorite child prodigy (alive and well to this date), was no longer as consumed[22] with producing

[22]While devoting himself to spreading the joy of mathematics among children, Pósa has

conjectures and proofs as he was at 13. Pósa actually "came back" briefly at the age of 23 to publish another highly influential paper (on Hamilton cycles in random graphs)[23].

When Erdős says someone "left us," he means the person passed away. He says "noise" for music and "poison" for alcohol. "The Book" is the collection of the simplest, most brilliant proofs of all mathematical theorems (possessed by God only, whether or not He/She[24] exists). Erdős has always had great appreciation for proofs that are "straight from the Book," himself having graced the mathematical literature with many gems that would qualify.

Some elements of his language strike the outsider for their seeming gender bias. He refers to women as "bosses" and men as "slaves." A "slave" may be "captured" (married) or "liberated" (divorced). Those who know him are not deceived by appearances; they know that Erdős is a genuine "equal opportunity" friend and collaborator who turns eagerly and with equal respect to anyone coming his way with a mathematical question, regardless of gender, religion, political persuasion, or other irrelevant factors.

In his paper on child prodigies [10], Erdős tells this story. "Lovász and Pósa, when they still went to high school, once asked me why are there so few girl mathematicians. I told them: suppose the slave children (boys) would be brought up with the idea that, if they are very clever, the bosses (girls) will not like them – would there be then many boys who do mathematics? Both said: well, perhaps not so many."

I witnessed a striking case of Erdős's evenhandedness in his collaboration with Imre Kátai, a number theorist at Eötvös University, much of whose work was inspired by early work of Erdős on additive functions. Kátai emerged as a strong political figure at Eötvös and became dean of the faculty of science around 1970. In the bitter infighting that divided the mathematical community at Eötvös from the mid-60s until the late 80s, Kátai's policies caused much grief to several of Erdős's friends. The losing side included Turán, Kátai's former mentor. While Erdős never shrank from taking sides, modestly but clearly, in this controversy, he continued to work with Kátai all those years and visited his office each time he arrived

continued to create mathematical gems but left most of them unpublished.

[23]With unfailing memory, Erdős points out that this account misses their last joint paper, written with Hajnal and appearing in the proceedings of Erdős's 60th birthday conference at Keszthely.

[24]One modern headache not facing speakers of Hungarian is the gender of third person pronouns; "he" and "she" are the same one-letter word "ő."

in Budapest.

Perhaps the most arcane element of Erdőshese is "S. F.," the term Erdős, the agnostic, uses for God. "The S. F. stole my passport," and "I'll travel once my cold gets better, which the S. F. saw fit to send me," he declares his misfortunes. Introduced in Erdős's vocabulary in the early 40s, the acronym stands for "supreme fascist," whose greatest offense perhaps is his/her reluctance to allow glimpses into the Book. "I don't mean to offend anyone. It is just an old joke of mine," Erdős says.

Erdős recalls that he recognized our (his) mortality at 4, and never recovered from the shock. He has been complaining of old age and stupidity for the past half century and has been commemorating this process with a growing string of letters. The first four of these, "P. G. O. M.," for "poor great old man," appear to have been around as long as his colleagues remember. Beginning with his age of 60, he has been adding a pair of letters every five years. At 65 he became "A. D." (an "archaeological discovery"), at 75 "C. D." ("counts dead," a reflection on the wise new rule adopted by the Hungarian Academy of Science that members over 75 do not count toward the quota on seats in the learned body, an age limit that since has been reduced to 70). He designed his epitaph: "Finally I am becoming stupider no more[25]," but this really only works in Hungarian, where such paramount concepts as "he is becoming more stupid" are expressed by a single, short word ("butul," pron. "boo-tool").

The making of disciples

> *Mathematicians become Collected Papers.*
>
> Paul Erdős, paraphrasing his mother's maxim

Joel Spencer emphasizes that he considers himself not as a *student* but as a *disciple* of Paul Erdős. Five quite different stories below may shed some light upon how this happened over and again[26].

Vera Sós

Vera Sós[27] does not really belong to the category of disciples. She should

[25] Végre nem butulok tovább.

[26] The stories of Spencer, Faudree and Chung are adapted from e-mail interviews conducted in summer, 1995.

[27] Her last name is pronounced "show-sh".

simply be called a friend. Her friendship to Erdős dates back much before their collaboration took off.

The younger daughter of a school teacher[28], Vera Sós was a student of Gallai[29] during the last four years of her studies at the Abonyi Street Jewish High School in Budapest. Gallai encouraged her obvious gift for mathematics. He gave her college texts to study (including the "little Knopp," the same book Erdős had studied with his father when he was in high school[30]), and introduced her to leading mathematicians Alfréd Rényi and Rózsa Péter.

Sós graduated from high school in 1948 and enrolled at Eötvös University as a mathematics and physics major. In December, Gallai introduced her to Erdős. It wasn't until 1956, however, that Sós and Erdős had their first substantial mathematical conversations. Meanwhile Sós became an instructor at Eötvös in 1950 (while still an undergraduate), graduated from Eötvös in 1952, enrolled in the graduate program and married Turán in the same year. She gave birth to their first child in 1953 (their second was born in 1960), and completed her graduate studies in complex analysis under Leopold Fejér between 1952 and 1955 with all but a dissertation. She studied various subjects (mostly analysis) with her colleague Tamás Kővári. One day Turán returned from a trip to Poland with a graph theoretic problem of K. Zarankiewicz and the three analysts, Kővári, Sós, and Turán "by accident" became the authors of a landmark paper in *extremal graph theory* (1954). The Polish connection was the cause of another accident two years later. On a visit to Poland Sós learned from Hugo Steinhaus about a problem in *diophantine approximation*, concerning the behaviour of the fractional parts of the numbers $n\alpha$, where α is an irrational number. It was love at first sight. Sitting in a café in Krakow, Sós proved her first result on the subject, and went on, with great speed, to building a substantial body of work around this concept, including her widely cited "3-distances theorem" (see e. g. Knuth: *The Art of Computer Programming*, Vol. 3, p. 511). The results were more than sufficient for a thesis. Her Ph. D. was awarded the same year. This subject has remained her favorite to this date.

From 1957 on, Erdős and *Anyuka* usually spent part of the summer at

[28]Promoted to school principal during the 1919 Commune, Vera Sós's father suffered the same fate as Erdős's mother during the right-wing retaliation after the Commune's fall (see the Time-line, 1920): he was forced to retire and supported his family by private tutoring and as substitute teacher.

[29]His former students rave about Gallai. Remarkably, Vera Sós was only one of three girls in that (all-girls) class who subsequently became mathematicians.

[30]See Time-line, 1929.

a resort of the Hungarian Academy of Science in Mátraháza in the Mátra mountains. Erdős was always surrounded by mathematicians with whom he enjoyed collaborating simultaneously on different subjects. The Turáns and the Rényis were among his frequent visitors at the hillside resort.

Sós did not actively pursue combinatorics in those years. Nevertheless, she was already one of the foremost experts in the area at Eötvös. In 1961, Rényi initiated a curriculum reform at Eötvös and suddenly there was room for more mathematics teaching to freshmen. Sós, by then a senior assistant professor, suggested that combinatorics be included in the curriculum, and she became the lecturer of the course for over two decades. Her first class included Béla Bollobás.

It was perhaps around this time that Sós got interested in delving more deeply into combinatorics. She began working together with Erdős and Rényi in earnest in 1962 on a visit to Mátraháza. The result was a joint paper on graphs of diameter two, which appeared in 1966. A few years later Sós introduced the concept of *Ramsey–Turán-type problems* and initiated a collaboration with Erdős on the subject. Their ground-breaking first paper appeared in 1970 and was followed by a series of papers on the subject a decade later.

Sós's energy and her dedication to the causes of the mathematical community have been extraordinary. She is unequalled in the constant attention she has devoted to promising students. In 1965 she initiated and launched the famous weekly Hajnal–Sós seminar at the Mathematical Institute of the Hungarian Academy of Science. For the past three decades, that seminar has been a major forum for new results in combinatorics, often made by young mathematicians, including undergraduate students. It is still going strong, even though Hajnal is absent. When in Budapest, Erdős usually attends the seminars, and not only to lecture there.

Since the mid sixties, Sós has been the driving force behind the periodic international conferences in combinatorics under the auspices of the Bolyai Society. Hungary was then a meeting place between East and West. Owing largely to Erdős's renown in the world, these conferences were highly successful, with large attendance by major combinatorists from overseas in spite of a total lack of funding. These meetings helped combinatorists from the Soviet bloc, especially Hungarians, build personal contacts with colleagues world wide at a time when travel to the West was extremely limited. Also, sale of the high quality conference proceedings helped balance the books of the Bolyai Society and allowed the Society to subsidize highly valued educational activities. Sós, along with Hajnal, constantly encouraged beginners

to participate in the meetings. Presentations by one or two undergraduates occurred nearly every time.

For over two decades, Sós has been one of Erdős's hosts whenever he visits Budapest. The two other principal hosts have been András Hajnal and especially Miklós Simonovits. Sós jokingly points out her handicap compared to her male colleagues; she unfortunately does not have the benefit of a helpful wife who would serve coffee while she does mathematics with Erdős.

The mathematical collaboration of Sós and Erdős became most intense in the 80s. Sós, often with Simonovits, made several important conceptual innovations during the 70s. She was later joined by Erdős in exploring the new concepts which include *Turán-Ramsey, anti-Ramsey, and structural intersection problems.* Since '85 Erdős and Sós have written a number of joint papers with Sárközy in *additive number theory.* To date, Erdős and Sós have nearly 30 joint papers.

Highly respected for her work both in combinatorics and in number theory, by 1980, Sós had risen to the rank of full professor at Eötvös university. In 1985, she was elected a member of the Hungarian Academy of Science and, in 1995, a foreign member of the Austrian Academy of Science.

András Hajnal

In 1955, on his first visit to Soviet-dominated Hungary after seven years of absence, Erdős met set theorist Géza Fodor at the lakeside vacation resort of the Hungarian Academy of Science in Balatonvilágos. Best known for his fundamental theorem on regressive functions[31] [18], Fodor, an assistant professor in Szeged, became Erdős's first Hungarian collaborator in set theory.

A year later Erdős visited Fodor in Szeged and met his "promising young colleague" *András Hajnal* (then 25), a graduate student of logician László Kalmár. Learning Hajnal's thesis topic, Erdős thought it was "too much logic" and asked Hajnal if he was also interested in *normal* set theory. Luckily, through Fodor's influence, Hajnal had already given thought to a problem on independent sets of a new type of set mappings. Hajnal generalized a question, originally raised by Turán in the thirties, on which both Erdős and Fodor had worked. The problem electrified their, until then polite, conversation. Then, as usual, Erdős proposed climbing the stairs of the tower of the local cathedral. (Szeged is situated in the plains, so climbing

[31] After Solovay, Fodor's result is usually referred to as the "pressing-down lemma."

mountain peaks was out of the question.) Half-way up, part of Hajnal's problem was solved. They continued through dinner at the Kalmárs. By the time Erdős left the next day, the foundations of a joint paper had been laid[32]. After much correspondence, the paper, entitled "On the structure of set-mappings" appeared in *Acta Mathematica Hungarica* in 1958.

There followed more than 50, often voluminous, joint papers by Erdős and Hajnal. They contributed to shaping *transfinite combinatorics,* now an integral part of set theory. Whenever it makes sense, Erdős and Hajnal investigate both the infinite and the finite versions of each question.

In the late fifties and early sixties, whenever Erdős was in Budapest, Hajnal would visit two or three times a week for intensive collaboration. They would work in *Anyuka*'s apartment, and Anyuka would serve coffee, keep Paul's papers in order, and leave the two mathematicians alone to work.

Hajnal also often visited Erdős in the Mátraháza resort of the Hungarian Academy of Science. As a junior scientist, he was not able to stay overnight at the resort, so he had to rent a room in the village. But he got "decent meals" on the Academy's premises, and had the pleasure of working with Erdős on set theory, simultaneously as Erdős was working with other mathematicians on different subjects [23].

While becoming the *number one* collaborator of Erdős, Hajnal also had the energy and devotion to serve the mathematical community in many ways.

As a professor at Eötvös University, he was very successful in popularizing set theory. His highly technical lectures were often attended by as many as 60 students.

In winter, 1965, fresh from a year's visit with Tarski at Berkeley, which Erdős had arranged, Hajnal assumed responsibility for an innovative part of the mathematics curriculum at Fazekas High School to the class that included Lovász and Pósa[33]. The same month, he joined Vera Sós in establishing their famous combinatorics seminar.

Hajnal became a senior researcher and later the director of the Mathematical Institute of the Hungarian Academy of Science. At the same time he was elected to the office of General Secretary and later President of the Bolyai Mathematical Society. Subsequently he became professor of math-

[32] A lively and detailed description of the events of that day is given by Hajnal in [23].

[33] See the "Child prodigies" section.

ematics at Rutgers University and the director of the DIMACS[34] research center located at Rutgers.

Once in a while, Hajnal would take a break and, leaving his official duties behind, he would join Erdős and *Anyuka* for three or four days in Mátraháza. Having achieved the requisite seniority, he would now stay inside the shrine. Occasionally he would bring his family along.

Whatever Hajnal's *office,* his office, telephone, desk, secretaries, and students would automatically belong to Erdős on *Pali bácsi's* increasingly frequent visits to Budapest. It seems likely that the same scenario now unfolds at Rutgers (with the possible exception of the status of the computer terminal).

Hajnal says he thought Erdős was quite old already in 1956 and often calls him Uncle Paul.

Their first joint paper became notorious for a gross oversight. A combination of two results proved in the paper easily imply that the first strongly inaccessible cardinal cannot be measurable. However, the coauthors did not observe this until after Tarski, building on results of Hanf, had given his own proof of this basic fact of the *theory of large cardinals* (1962) (cf. [23]). "The Tarski-Hanf proof was better, it reached farther," comments Erdős in 1995.

Joel Spencer

Joel Spencer left Harvard in 1967 with all but a dissertation. He was at Bell Laboratories in 67-68 and heard much about Erdős from Ron Graham and others. Spencer continued in '68 at Rand Corporation in Santa Monica, California, near UCLA, a frequent stop of Erdős, who had been working there with Ernst Straus, Theodore Motzkin, and others. Spencer saw Erdős sometime around then at a conference, but if they spoke, it was only a few words.

Then in 1969, Spencer managed to show that any tournament has a ranking with "fit" at least $cn^{3/2}$, resolving a question of Erdős. The two met at a hotel near the UCLA campus where Erdős was staying. Erdős immediately recognized how the method would apply to discrepancies in colorings. The results ended up as two papers in *Networks* in 1972, one by Spencer alone, the other as his first joint paper with Erdős. These results become the core of his dissertation, which Spencer completed in 1970.

"By April 1970 my contact with Paul and my courage were sufficient that we invited him to our apartment for what is for me a most memorable

[34]DIMACS: Center for Discrete Mathematics and Computer Science.

'five generation' dinner. Paul's mother and Paul were there as was Ray Fulkerson and then Maryann and I and then David in utero, and thus I'm quite sure of that date!", Spencer recalls.

"About when I finished the dissertation I asked Paul if we could write a book together on the *probabilistic method.* Looking back I can't understand how I had the audacity to ask. To my delight and surprise Paul said yes. At that time the general combinatorial audience liked Paul's results in this area very much but he was really a 'voice in the wilderness', there were very few others working on probabilistic methods, it was like a magical idea.

"The book took about two years [to write]. Basically the writing was all mine and the theorems (with a couple of exceptions) were Paul's. Paul would bring up an area and point out the key results and the interesting open problems and I'd try to write them down."

"Probabilistic Methods in Combinatorics" appeared in 1974 and was an instant success. At the same time, Spencer undertook the editing of the selected works of Erdős in finite combinatorics. The volume appeared in 1973 under the fitting title *"The Art of Counting"* [11] and became an invaluable reference. Unfortunately, it has been out of print for over a decade.

This effort put Spencer in contact with many Hungarian mathematicians. For instance, he obtained a foreword to "The Art of Counting" from Turán. "I felt even before I got to know many Hungarians (besides Paul) that I was working on 'Hungarian mathematics'."

Spencer first visited Hungary in 1973 on the occasion of the conference in Keszthely celebrating Erdős's 60th birthday. Spencer returned with his family for a full academic year, 1975-76, on a National Science Foundation exchange. His son David was in first grade at the British Embassy school and his daughter Danielle went to a French nursery school. By the end of the academic year Spencer surprised his friends with seminar lectures in fluent Hungarian (spiced with some lovely terminological innovations). For an American with family, spending a year in a Soviet-bloc country must have been a big decision. "By '76, the curtain had lifted a little for mathematicians but not by much. As for why we went: going 'behind the Iron Curtain' was certainly an exciting adventure but the main thing was Budapest being, for me, a mathematical Mecca. As indeed it was."

Spencer visited again for three months in '83 and made many other visits of shorter duration. These trips did not serve to see Erdős who was not in Hungary particularly more often than he was anywhere else. Erdős frequently visited Stony Brook, Spencer's hometown from 1975, and often stayed at Spencer's home. "Generally those visits were quite short, a couple

of days at a time. We've met in many other places. Budapest certainly, also New Jersey frequently, and at many, many meetings, such as Boca Raton[35]."

Ralph Faudree

An accomplished group theorist with some interest but little experience in combinatorics, Faudree was alerted by colleagues at Vanderbilt University that Paul Erdős was there, "as well as a young mathematician by the name of Lovász." So Faudree, then an associate professor at Memphis State University, took to the road and drove 200 miles to Nashville to hear the P. G. O. M.

At the evening party he showed Erdős a paper he had just completed with Richard Schelp on Ramsey numbers of pairs of cycles (his first paper outside group theory). Erdős glanced over the paper and said, "Maybe I could come to Memphis for a visit and we could do some mathematics." Faudree "floated all of the way back to Memphis" after this remark; it marked the beginning of a long friendship and collaboration in *generalized Ramsey theory*.

"One interesting incident that happened that first evening was that there was a table-tennis table at the house and Paul was playing and beating various people. I asked to play him a game, and he remarked that it was late but he could play one game, which we did. He lost, and being the competitive person that he is, he suggested that we play another game, etc.

"This was a banner day for me, I met Paul Erdős, he read my mathematics, we took the first step on a long friendship, and I beat him in table tennis."

Erdős paid his first visit to Memphis a year later. The result was his first joint paper with the Memphis trio of Faudree, Schelp, and Cecil Rousseau on multicolor Ramsey problems (*Journal of Combinatorial Theory*, 1976). Having found his home in combinatorics, Faudree never returned to the theory of groups.

Erdős became an Adjunct Professor at Memphis State University (now University of Memphis) in 1975 stipulating that he visit for one week each year. Typically, he visits several times a year and spends over a month in Memphis, mostly as a house-guest of the Faudree family. There is a bedroom for his use in the Faudrees' home, where he also leaves some of his things while away. "My children grew up with Uncle Paul," Faudree says with affection.

[35] *Boca Raton, Florida:* frequent venue of an annual combinatorics conference featuring Erdős each year among its invited speakers. See Time-line, 1970.

Like Spencer, Faudree first visited Hungary for the 1973 conference in honor of Erdős's birthday (which, incidentally, was chaired by Hajnal; this author served as the secretary of the organizing committee). With plenty of encouragement from Erdős, Faudree decided to spend the academic year 1980-81 in Budapest with his family.

"Paul was very careful to see that his friends in Budapest 'took care of me,' which they certainly did. My daughters were 12 and 13. It was a great trip for the entire family. The girls attended the U. S. Embassy school. Also, they went to an after school language program with Hungarian students. With the public transportation and the safe streets, they had the freedom to travel all over the city by themselves, they took ballet, and violin lessons etc. on their own." The entire family enjoyed the experience and returned for another academic year in 1990.

By now, Faudree has over 35 joint papers with Erdős, and the number is growing. "Paul Erdős is my combinatorial mentor. I had no formal education in combinatorics. I barely knew what a graph was when I met Paul, and I certainly knew nothing about asymptotic results, probabilistic techniques, etc. As he has done so often with many people, he took me as a young mathematician under his wing – he taught me and introduced me to other active researchers. I can now ask better questions and have more mathematical tools and techniques because of the exposure to Paul.

"He comes to my house not just as a mathematician but as a friend of the family. We have celebrated Erdős's 'birthday wakes'[36] many times in Memphis and also in Budapest. My family knows that they have had the opportunity to know in a closeup and personal way a person of truly exceptional ability and an interesting character, but also such a decent and kind human being."

Fan R. K. Chung

Fan Chung missed that famous 1973 conference. She received her undergraduate degree in Taiwan in 1970 and her Ph. D. at the University of Pennsylvania four years later. Working in Ramsey theory under the direction of Herb Wilf, she heard much about Paul Erdős during her student years. But she did not meet Erdős until she joined the group of mathematicians headed by Ron Graham at Bell Laboratories in Murray Hill, N. J. in 1974, right after graduation. By then, Graham was one of Erdős's closest associates in the U. S., and Bell Labs became one of Erdős's frequent stops. Energetic and

[36] *Birthday wakes:* Erdős's joke reflecting his dim view on aging.

ambitious, Chung did not hesitate to seize the opportunity. She reminisces of their early, and particularly active, years of collaboration:

"It was wonderful to work with Paul. He just kept asking questions and pushing the limit of what can be done. His questions serve as a guide which usually leads to the right directions. Even when I thought that we have found the answers to the questions and perhaps deserved a rest, Paul would not stop and raised new questions. It was particularly nice when we worked on unavoidable hypergraphs as Paul's intuition cut through the complicated problems. We still have a big folder with many unresolved questions on this topic. There is no doubt that Paul asserted a great deal of influence on that stage of my research. Just like the style of Paul in playing the game Go, Paul enjoys battles in close range in mathematics as well."

In a major event of U. S. corporate history, Bell Telephone Company was split up in 1983. Bellcore, a counterpart to AT&T Bell Laboratories was established to serve the basic research needs of the regional Bell service companies. Chung became the founding manager of the Discrete Mathematics research group at Bellcore while Graham stayed with AT&T.

Soon after the corporate divorce, Graham and Chung married and created a "permanent" home for Paul Erdős in New Jersey. In 1989 they made a major improvement to their home, adding a separate "Erdős Room" to it.

"When Paul is in town, it is essential to arrange for Paul-sitting. In other words, Paul enjoys to be constantly surrounded by mathematicians. In fact, it is more than an enjoyment; it is a necessity. Usually, Paul stays with us when he is around. So we have a stream of mathematicians coming and going, talking and writing.

"Paul enjoyed talking to my daughter Laura before I put her to bed when she was an *epsilon*. We also enjoy playing ping-pong. He is surprisingly good at it, he has an excellent blocking instinct which translates to good defense. Most people know that Ron[37] is very good at ping-pong but they don't know the story behind it. Many years ago when Ron first played ping-pong with Paul, Ron lost. After that defeat, Ron got serious at ping-pong and became New Jersey Champion. He still plays with Paul, giving him a handicap."

[37]Graham's physique is just the opposite of Erdős's. Six-foot-two tall, athletic, he does triple somersaults forward and backward on the trampoline (there is one in his home). He was California State champion in 1960 and performed briefly in a circus trio called Bouncing Bears. He juggles six balls and loves to teach mathematicians to juggle (cf. [5]). He has several apprentices among Hungarian combinatorists. The foremost one, Péter Frankl, a powerful and prolific mathematician conversant in seven languages, juggles seven balls, and makes the greater part of his living performing for Tokyo's street crowds.

Chung quickly moved up the Bellcore hierarchy and at the same time became a leader in the graph theory community; for five years, she was the Editor-in-Chief of the *Journal of Graph Theory*. After holding visiting professorships at Princeton and Harvard, she left Bellcore in 1994 and settled at her alma mater, the University of Pennsylvania, as a distinguished professor.

The "Erdős Room" may no longer be as frequently used as originally intended. Graham mentions that due to his and Chung's frequent travels, Erdős, when visiting New Jersey, now often stays with C. U. N. Y. professor Melvyn Nathanson. This, however, has become more complicated lately since Nathanson has two small and energetic children.

Historical background: pre–WW I

Paul Erdős was born in Budapest, Hungary, on March 26, 1913. Within a year and a half, the world erupted in flames. For the bigger part of this century, a grim history was a determinant factor of the destinies of the millions in Central and Eastern Europe. Jewish descent was soon to become a cause for humiliation, and for physical annihilation a quarter of a century later.

Erdős grew up in that world, so a brief historical background seems to be in order before we turn to our time-line, an account of random events and encounters that shaped the path of Erdős and the legends around him. In a few sections to follow, we summarize some of the relevant events in the history of Hungary, Hungarian Jews, and mathematics in Hungary.

The Habsburg Empire was transformed into the "dual monarchy" of Austria-Hungary in the *Ausgleich* (Compromise) of 1867 between emperor Franz Josef and Hungarian politicians. After centuries of Austrian domination, a Hungarian government was set up, independent of Austria in all matters except foreign policy, defense, and "joint finances." A third of a century of unprecedented growth ensued, belatedly but rapidly ushering in the age of capitalism to Hungary. At the same time, political power remained in the hands of the traditional aristocratic-gentry classes, who were insensitive to the grave social problems of the country and hostile to the aspirations for self-determination of the non-Hungarian minorities within Hungary (nearly half the population).

It was during this era that Budapest grew into the busy and inviting metropolis that we now know, a "little Paris on the Danube."

One of the first acts of the liberal government of 1867 was the full eman-

cipation of the Jews, opening up all "civic and political functions" to them. Hungarian Jewry, growing rapidly after the *Ausgleich* and numbering about 900,000 at the time of the 1910 census, responded with enthusiasm and went on to make immense contributions to Hungary's progress and modernization, including commerce, banking, the development of competitive modern industries, the professions, as well as to all aspects of intellectual life – sciences, literature, performing arts. The Jewish contribution to the mathematical culture of Hungary was particularly eminent[38].

The Jewish assimilation movement, whose roots can be traced as far back as 1844, gained strong momentum after the emancipation act of 1867. It was wholeheartedly embraced by the Neolog (reformist) majority of the Jewish community in Hungary, especially in Budapest and within the boundaries of what came to be defined as Hungary under Trianon[39].

During this "Golden Era" of Hungarian Jewry, the overwhelming majority of the Jews of multinational and multilingual pre-WW I Hungary adopted Hungarian as their language of communication. Many (including Erdős's father) changed their German-sounding names to Hungarian ones and abandoned Jewish customs. Many Jewish community leaders became ardent patriots, supporters of Hungarian nationalistic aspirations.

KöMaL: the Mathematical and Physical Monthly for Secondary Schools

It was love at first sight.

László Lovász about KöMaL which he first saw as an 8th-grader in 1962

In a remarkable display of vision, dedication, and leadership, high-school teacher Dániel Arany (1863 - 1945) founded *Középiskolai Matematikai Lapok*, a high school mathematical monthly. We shall refer to the journal as KöMaL, its currently popular acronym. KöMaL is a unique forum that invites mathematically gifted high school students to an ongoing creative problem solving competition. Students send their solutions to the editor by mail every month. The solutions are graded by dedicated volunteers. The best solutions are printed in the journal and credited to the solver, and all solvers are listed.

[38] Cf. in particular the next section.

[39] The current boundaries of Hungary were defined by the Trianon peace treaties after WW I. Cf. the section "Peace á la Versailles."

In 1896, high school teacher László Rátz (1863-1930) took over[40] as chief editor; Arany continued to contribute enthusiastically. In 1904, a physics section was added to the journal.

The publication of KöMaL was interrupted by WW I with the drafting of Rátz to combat duty. It resumed in 1925 (with physics added to the title of the journal), under the energetic and resourceful editorship of another dedicated teacher, Andor Faragó (1877-1944), a friend of Erdős's parents from their college years. Since 1926, KöMaL has been publishing the photos of its most successful problem solvers every year.

Faragó remained the editor until 1939 when the journal was shut down again, to resume in 1947 under János Surányi, a friend and collaborator[41] of Erdős.

KöMaL has nurtured mathematical talent and shaped the community of young mathematicians for over a century. Through the journal, generations of eager youths have learned each other's names, appreciated each other's thoughts, and matched their prowess in a common pursuit. The journal, and the community constantly building around it, has often been given a large share of credit for Hungary's success in mathematics [31, 32].

Erdős was a diligent problem solver, his photo appearing each of his high school years (1927 to 1930). Other devoted problem solvers include Tibor Gallai (Grünwald), György (George) Szekeres, Eszter Klein, Pál Turán, and Márta Wachsberger. These are just a few of the peers of whom Erdős learned through KöMaL, and who subsequently became his lifelong friends.

Turán, three years Erdős's senior, first met Erdős in 1930 at Pázmány (now Eötvös) University in Budapest when Erdős was a freshman. But, as Turán recalls with pride [41], their first "joint work" appeared in print two or three years earlier; it was a solution to a KöMaL problem which they both had found (and which no one else did).

Ever since graduation, Erdős has remained faithful to the journal. He proposes problems, writes articles. In the centennial issue of KöMaL [31, 1993/10, p.486], László Lovász recalls:

[40] At the beginning of WW I, the high school classes of Rátz included 11-year-old John von Neumann and 12-year-old Eugene Wigner. Half a century later they remembered Rátz as a "miraculous" teacher [32, pp. 20-22].

[41] The first serious mathematical reading of this author (at 16) was the enchanting "Selected Topics in Number Theory" by Erdős and Surányi, opening up for him the delightful world of big-O's, log log-s, conjectures, asymptotic upper and lower bounds and the tantalizing gaps between them ... After four decades, an English edition is finally forthcoming [16].

> "KöMaL was much more than a competition. In one of the first issues I got in my hand, there was an article by Paul Erdős on combinatorial geometry. I was surprised and excited to learn that I could understand what great mathematicians think about and how many beautiful and hard unsolved problems one can see just looking around, even in a classical field like geometry." [Lovász was an eighth-grader at the time.] "I read the article at least twenty times. The articles and lectures of Erdős still have this mesmerizing effect on their audiences. I am grateful to KöMaL for conveying this to a wide circle of students."

Hungarian emigrés, formerly enthusiastic KöMaL affiliates, have made various attempts to plant the seeds in different parts of the world. Martha Sved (Wachsberger) reports in the centennial issue of KöMaL [31, 1994/4, p.154]:

> "Both Esther [Klein] and I were regular problem solvers of KöMaL. ... My husband George Svéd (Schossberger) was unaware of Faragó's journal, the mathematics teacher in their school did not promote it, but he won the Eötvös competition in 1928. We have been living in Australia for nearly fifty years with George Szekeres and his wife Esther Klein. We have been trying to produce a similar mathematics journal here for the students. Within a more modest framework we have succeeded. Since the distances are very large here, each state now has their own journal: 'Parabola' in New South Wales, 'Trigon' in Southern Australia."

George Berzsenyi has been tireless at nurturing young mathematical talent in the U. S. via a number of programs such as the U. S. A. Mathematical Talent Search, COMAP (Consortium for Mathematics and Its Applications) and its newsletter, "Consortium" [8], and the high school mathematics and physics journal "Quantum," an adaptation of the Russian "Kvant." In his recent venture "Mathematics and Informatics Quarterly," resembling the format and contents of KöMaL and edited jointly with the Bulgarian Academy of Sciences, Berzsenyi reflects on the 100-year history of KöMaL in his Editorial [33, 3/94, pp. 103-105].

> "It is my hope that we will all be encouraged by the example of this small country's unbelievable success, not just in mathematics, but in all fields of science and engineering. ... there is an editorial board of outstanding mathematicians in charge of KöMaL;

> since most of them *grew up on KöMaL*[42], they willingly devote much time and attention to its continued excellence. ... With the M&IQ we hope to internationalize the example given by KöMaL."

It is a tragic note to Hungarian history that Andor Faragó, this selfless and singularly effective contributor to one of the prized and long lasting intellectual assets of the nation, perished[43], together with his two sons, under abominable circumstances at the hands of *Nyilas* (Hungarian Nazi) gangs in 1944. Arany, the founder of KöMaL, was forced into the Ghetto in 1944, at which point he donated his valuable library to the University. He survived the persecution and the war, but only by a few months, for he died shortly after liberation.

Historical background: peace á la Versailles

> *You may do as you wish but I will remain the way I was born.*
>
> Six-year-old Paul Erdős on his mother's suggestion to get baptized in order to avoid persecution.

The backdrop to the experience of Erdős in his childhood and early youth was the aftermath of World War I.

More devastating to Hungary than the ravages of the lost war was the subsequent peace treaty. After the war, Austria-Hungary disintegrated. The independent Hungarian Republic, declared in October 1918, faced internal chaos and foreign invasion. The short-sightedness of the Western powers contributed to the collapse of the progressive democratic-socialist Károlyi government, tossing the country into the hands of extreme radical regimes.

Communists took over from Károlyi on March 21, 1919 and declared the "Hungarian Commune,"[44] modeled after Soviet Russia. The Commune

[42] Emphasis mine – L. B.

[43] No photo of Faragó could be located for the centennial issue of KöMaL. This is especially ironic for the man who created the invaluable photo archive that preserved high-school age photos of Erdős and his colleagues for posterity.

[44] The official name was "Hungarian Council Republic," the word "Council" being the translation of the Russian word "Soviet" which refers to the workers' and peasants' councils ostensibly holding all power.

lasted 133 days and collapsed on August 2, 1919. Rumanian troops occupied and looted the country.

The short-lived Commune's acts ranged from the secularization of education and the separation of church and state (assisted by a good number of outstanding intellectuals of the country) to nationalization of the economy and culture to "Red Terror," the bloody suppression of opposition to their insensitive doctrinaire Marxist regime. The subsequent "White Terror" "dwarfed in ferocity and magnitude the Red excesses" [4, Vol.1,p.16]. Death squads roamed the country slaughtering leftists and their alleged sympathizers, including a disproportionate number of Jews. Anti-Semitism was on a sharp rise.

The peace treaty signed on June 4, 1920 in the Trianon palace in Versailles reduced historic Hungary to less than a third of its territory. Two-thirds of its population, including a third of its ethnic Hungarians, came under the rule of the successor states.[45]

The public in reduced Hungary reacted with despair and extreme nationalism. Jews became the scapegoats.

Erdős says he "never cared about being Jewish." This statement, strange to the ears of many in our age of cherished traditions and roots, is consistent with the assimilation movement of the "Golden Era."

If many Hungarian Jews wanted to be like everyone else in the country, post-WW I society harshly reminded them that they were not. Perpetrated by "fraternal associations" ("bajtársi egyesületek"), from 1920 on Jews were regularly beaten at the universities. Even in the streets, Jews were frequently assaulted after the fall of the Commune. Erdős and his mother witnessed such scenes from the balcony of their 5th floor apartment[46]. One day his mother asked Paul whether they should perhaps get baptized to avoid such atrocities. "You may do as you wish but I will remain the way I was born," the six year old replied.

[45] The main beneficiaries were newly created Czechoslovakia and Yugoslavia and a greatly enlarged Romania. Acts of ethnic intolerance by the new masters began even before the conclusion of the peace treaties and included the expulsion from (greater) Romania in the summer and autumn of 1919 of the entire ethnic Hungarian faculty of the University of Kolozsvár (Cluj). Kolozsvár's was one of the foremost universities of pre-war Hungary. Its mathematics faculty had included Leopold Fejér, Frederick Riesz and Alfréd Haar, textbook names today. Similar was the fate of Elisabeth University, Pozsony (Bratislava, Slovakia). Much of the refugee faculty, including Riesz and Haar, subsequently settled in the city of Szeged and created a world class university there in 1921.

[46] 14 Kálvária tér, Budapest.

The "Numerus Clausus" law, enacted in September 1920, limited Jewish admissions to universities at 6%. It was the first major anti-Jewish legislation in post-WW I Europe. The infamous law was somewhat relaxed in 1928. As a result, Erdős, Gallai, Turán, and other winners of national competitions were allowed to enter the university without examination. (However, their admission triggered a fierce campaign from extreme right-wing newspapers.)

Anonymous

L. A. is studying Jordan's theorem.

Paul Erdős, informing his friends that another friend, László Alpár, active in an underground organization, had been arrested and was observing the difference between the interior and the exterior of the wall of a jail cell.

There is a picturesque castle in Budapest's City Park. In the courtyard of the castle sits the hooded bronze statue of *Anonymous*, chronicler of the Hungarian kings of the late 12th century.

In summers in the early '30s, the statue of *Anonymous* became the weekly gathering place of a close circle of remarkable young mathematicians, foremost among them Erdős, Gallai, and Turán. Largely the same group made excursions every Sunday, year-round, in the charming hills around Budapest. In this circle, "friendships were forged which became the most lasting that I have ever known and which outlived the upheavals of the thirties, a vicious world war and our scattering to the four corners of the world," writes George Szekeres, then a student of chemical engineering and budding mathematician, in his "Reminiscences" [37], a foreword to the selected papers of Erdős [11]. In another forword [41], Turán testifies: "The main subject of conversations was mathematics; since Gallai and Erdős attended the graph theoretic lectures of Dénes König, graph theory was discussed often."

Szekeres offers a warm and lively account of the mathematical excitement generated within this circle of mostly Jewish students, to whom society had offered the bleakest of prospects, humiliation, fear, and certainly no hope of academic employment. Szekeres describes how, through interaction within the *Anonymous* group, Erdős reached what seems, by all accounts, a decisive stage in his combinatorial studies.

Around 1932, Esther Klein proposed a curious problem in plane geometry: among any five points in the plane, there are four that form a convex quadrilateral. With Erdős, they immediately generalized the problem to n-gons. Szekeres, excited by the problem as well as by its originator, made a great effort to be the first to solve it. He succeeded after a few weeks, combining Klein's observation with Ramsey's Theorem, which he had rediscovered for the occasion. Erdős then found a direct proof, yielding a more realistic bound. The two proofs were combined in a now classic paper by Erdős and Szekeres which appeared in 1935 in the *Compositio Mathematica.* (Erdős recalls the name of the editor, Reinhold Baer, handling the paper.) Finally, Szekeres married Klein in 1936, hence the phrase "Happy End Problem," coined by Erdős.

Although discovered by Ramsey and rediscovered by Szekeres, it was Erdős who realized the vast domain behind Ramsey's theorem, waiting to be explored. An area critical to both finite and transfinite combinatorics, Ramsey Theory has been one of the major creations of Paul Erdős. At the same time, the Happy End Problem also seems to have triggered another insight of Erdős, the recognition of how much combinatorics there is in discrete geometry, a discovery that has given new life and direction to elementary geometry.

Historical background: the Holocaust in Hungary

> *. . . on the very eve of triumph over the barbarism . . .*
>
> President Roosevelt on March 24, 1944, warning of the imminent annihilation of Hungarian Jewry, just a year away from Allied victory, in the wake of the German invasion of Hungary.

In WW II, Hungary fought on Hitler's side. Paradoxically, this circumstance offered some temporary protection to Hungarian Jews. Increasingly oppressive laws limited their employment, the men were drafted to "labour service companies" where they were often subjected to abuse, but the *en masse* deportations to Auschwitz and the death marches had to wait a few years.

Hitler's "Final Solution" caught up with Hungarian Jewry on March 19, 1944, when the Third Reich felt compelled to take direct control of Hungary, an ally, by unopposed invasion. With the invading force arrived

Adolf Eichmann and his meticulous staff, and within four months, virtually the entire Jewish population of Hungary outside Budapest, nearly half a million people[47], were on their way to the gas chambers. The barbarity and speed of the destruction of the Jewry of the Hungarian provinces has been characterized by Churchill[48] as "probably the greatest and most horrible crime ever committed in the history of the world"[4, Vol.2, p. 1111].

Although the systematic death transports failed to reach Budapest Jewry, the losses in the capital, too, were staggering. The *Nyilas* (Arrow-cross, Hungarian Nazi) coup of October 15, 1944, unleashed an unchecked terror of sadistic *Nyilas* gangs, tormenting and butchering tens of thousands of Jews in the town or on westward death marches. By the combination of miracle, logistic problems, the progress of war, as well as the heroism and cunning of Swedish diplomat Raoul Wallenberg, the bigger part of the 246,000 Jews of Budapest[49], about 144,000 people, survived. Excluding the USSR, it became by far the largest Jewish community in postwar Europe.

Advancing Soviet troops encircled the capital by the end of December and liberated Pest[50], including the Ghetto, on January 17, and Buda on February 13. By then, over 100,000, or 40%, of Budapest Jews had been murdered. In the countryside of Trianon Hungary as well as in the ceded areas, the loss was over 80%.

Among those who perished were: four out of five siblings of Erdős's mother and the husband of her only surviving sister; several of Erdős's close

[47] The Jewish population of the Hungarian countryside at the beginning of the German occupation was 530,000. This includes the areas outside Trianon Hungary ceded to the country in 1938-40. Between May 15 and July 8, 1944, 440,000 of these people, or 83%, were deported to Auschwitz. Many close relatives of Erdős lived in the ceded areas.

[48] Churchill's strong words and Roosevelt's similar statements should be contrasted with their governments' callousness towards the plight of European Jews during the war. Although accurately informed by mid-1942 from multiple corroborated reports of the realities of Auschwitz, the Allies did nothing to frustrate the Nazi genocide machine, not even extending a helping hand to those trying to flee. The British stuck to the end to their 1939 "White Paper" that limited immigration to Palestine to 75,000 during 1939-44. The U.S., too, steadfastly refused to liberalize their immigration policies. The Allies consistently rejected any rescue schemes placed before them. A report entitled "... on the Acquiescence of This Government in the Murder of the Jews," was presented by the U.S. Treasury Department to President Roosevelt on January 16, 1944. The President was "shocked," yet the inaction largely continued. The Allies resolutely refused suggestions to bomb the rail lines leading to Auschwitz and the crematoria even in summer, 1944, at a time when they carried out massive air strikes on factory areas near Auschwitz. [4, pp. 1095-1118].

[49] The figures are based on [4, vol. 2, p. 607 and pp. 1143-1144]. The numbers refer to those identified as racial Jews under the anti-Semitic legislation of 1941.

[50] Pest: the part of Budapest to the East of the Danube river. The other part is Buda.

friends, including Géza Grünwald and Dezső Lázár, promising young mathematicians from the *Anonymous* group; and tireless KöMaL editor Andor Faragó with his sons. Dénes König, who had introduced Erdős and the world to graph theory [27], committed suicide when ordered by the janitor to move to the Ghetto.

Parents and grandparents of Paul Erdős

We preface our Erdős chronology with some family history.

Erdős's paternal grandfather was a school teacher in the town of Hódmezővásárhely, Hungary, and died in 1914. Erdős's paternal grandmother died two years later. They had four sons; one of them was killed in action in WW I.

Erdős's maternal grandfather owned a small shop in Vágbeszterce, Hungary [Považská Bystrica, Slovakia]. He died at a young age in 1905. Erdős's maternal grandmother, the only grandparent with whom Erdős had any contact, lived in Vágbeszterce/Považská Bystrica until her death in 1944. Four of their six children were murdered by the Nazis.

Erdős's *father*, Lajos Erdős (born Lajos Engländer), was born in Hódmezővásárhely, January 30, 1879, and died in Budapest, August 1942.

Erdős's *mother*, Anna Wilhelm was born in Vágbeszterce, July 6, 1880, and died in Calgary, Canada, January 1971.

Married in 1905, both of Erdős's parents were high school mathematics teachers. They met as students at Pázmány (now Eötvös) University in Budapest[51]. They were contemporaries of the renowned mathematicians Tódor Kármán (Theodore von Karman) and Lipót (Leopold) Fejér and had known them from college. Kármán was fond of "blue-eyed Anna Wilhelm," and she bequeathed his affectionate words to her son's memory.

Erdős remembers his parents with great affection. Even today, he refers to them as *Anyuka* (mommy) and *Apuka*[52] (daddy). He received much of his education from them.

Erdős's grandparents were observant Jews but his parents did not practice the religion. When Lajos Erdős visited his fiancée on Yom Kippur, a

[51] Women were first admitted to Pázmány University in 1896.

[52] Erdős consistently uses these affectionate terms, but only in Hungarian. In English he says "my mother/father."

day traditionally devoted to introspection, he found her fasting *and* reading Maupassant. He pointed out the paradox he sensed in this combination. She agreed. Shedding tears, Anna relinquished the old traditions.

Apuka was interested in philosophy and occasionally wrote[53] for "Huszadik Század" ("Twentieth Century"), a monthly "Review of social science and social policy" of progressive left-wing intellectuals (1900–1919).

Anyuka lived in Budapest through Nazi persecution and communist paranoia. At the age of 84, she joined her son on his endless tour of the globe. From then on until her sudden death in Calgary at the age of 91, she was with him at all times. Meanwhile she became conversant in her fourth language, English.

TIME-LINE

1913. Paul Erdős is born on March 26, 1913, in Budapest, Hungary (part of Austria-Hungary at the time).

    Paul's birth is marred by tragedy. His mother leaves two healthy daughters, Magda (5) and Klára (3) at home when she goes to the maternity ward to deliver Paul. By the time mother and infant return home a few days later, both girls are dead. They contracted septic scarlet fever and succumbed within a day. Paul grows up as an only child. Although his mother seldom mentions his sisters, this trauma obviously affects Paul's upbringing.

    Anyuka does all she can to protect Paul from exposure to disease. Paul will contract scarlet fever at age 23, whooping cough at 30, and chicken pox at 34.

1914. July 28: Austria-Hungary declares war on Serbia. Russia, in turn, declares war on Austria-Hungary. Within days, Germany, France, and the U. K. join in: WW I begins. By the end of August, large numbers of Hungarian men are slaughtered in the first battles or are taken prisoner by the Russians. Long forced marches of the often wounded prisoners are common. Lajos Erdős is captured by the Russians in August and is taken as POW to Siberia for *six years.*

[53] One of his articles, an obituary for Henri Poincaré extolling Poincaré's genius in the style of the time and reviewing his philosophy in some detail, appeared in 1912 [9].

1916. Paul is greatly motivated to learn how to count. He plays with the calendar and knows how to count the days remaining till holidays or vacations months ahead when he would see more of *Anyuka* and less of the German *Fräulein*[54] [10].

1917. By the age of four, Paul multiplies 4-digit numbers in his head. This is the year of what he calls his "first independent discovery," the negative numbers: he reports to *Anyuka* that "if you take away 250 from 100, you get 150 below zero." Another discovery: Paul recognizes that we all die, a thought that has never since allowed him to rest.

1917. Lenin's Bolshevik revolution takes place in Russia.

1918. The "Bulgarian front" collapses, Austria-Hungary is defeated and dissolved. The Hungarian Republic is declared. Hungary is invaded from all sides.

1919. Communist takeover takes place in Hungary on March 21, 1919 (the Hungarian Commune). The regime collapses on August 2. "White terror" begins, anti-Semitism rises.

From the balcony of their apartment, Erdős and his mother witness assaults on Jews in the street.

1920-45. Right-wing governments rule Hungary.

1920. The peace treaties of Trianon dismember Hungary, inducing an influx of masses of ethnic Hungarian refugees from the successor states, especially from Transylvania, which is annexed by Romania. Public despair rises over the nation's catastrophe. Jews are the scapegoats.

Numerus Clausus act limits Jewish admission to universities at 6%.

1920. *Anyuka* was promoted to school principal during the Commune; for this "crime" she is permanently barred from teaching at public schools. She contributes to their living by private tutoring; the family maintains a reasonable middle-class living.

Surviving malnutrition, extreme cold, and civil war, *Apuka* returns from Russia in November. Little Paul greets him: "*Apuka*, you are *really* old!"

[54] *Fräulein:* Governess.

1919-24. Paul does not attend school, partly because he has difficulty adjusting to discipline, partly for his parents' fear of contagious diseases. He receives private tutoring and learns a lot from his parents. *Apuka* teaches him English, and they both teach him mathematics.

1923. At 10, Paul learns from *Apuka* that there are arbitrarily large gaps between consecutive prime numbers. Paul knows he will be a *mathematician* or an *astronomer.*

1924-26. For the first time, Paul enrolls in a school. He attends the 6th and 7th grades at the Gimnázium (High School) of Tavaszmező street. He goes back to private tutoring for the 8th grade.

1925. "At 12 I knew that eventually I'd have to leave Hungary because I am a Jew," Erdős says. From 1920 on, beating Jews has been a regular occurrence at the universities.

1926-30. For two out of these four school-years (9th through 12th grades) Erdős attends Szent István (Saint Stephen) High School where *Apuka* teaches. Erdős completes the rest by private tutoring and graduates in 1930. Throughout these years, he is a diligent problem solver of KöMaL, the high school mathematical and physical monthly edited by Andor Faragó. Erdős's photo is published in 1927 and in each subsequent year among those of the best problem solvers.

Neither Erdős nor his fellow KöMaL problem solver Paul Turán fare particularly well at the national Eötvös mathematical competitions (a creative problem solving exam for fresh high school graduates, see [25]); Gallai wins the first prize in 1930. Gallai is first denied admission to the university because of the *Numerus Clausus* but is admitted half a year later for having won the Eötvös test[55].

[55]Organized by the Hungarian Mathematical and Physical Society, the annual Eötvös mathematical competition was first held in autumn, 1894. Every fresh high school graduate was eligible to attend. Among those mentioned in this article, the following had been winners of the Eötvös competition: Lipót Fejér (2nd prize, 1897), Tódor Kármán (1898), Dénes König (1902), Alfréd Haar (1903), László Kalmár (1922), György Svéd (Schossberger) (1928), Tibor Gallai (Grünwald) (1930), László Alpár (1932), Endre Vázsonyi (Weiszfeld) (1934). After WW II, the test was renamed "Kürschák competition." Eligibility was extended to high school students so the particularly gifted could attend several times. Winners included Béla Bollobás (2nd prizes, 1958 and 1961), Attila Máté (1962, 1963), László Lovász (shared 2nd prizes, 1964 and 1965), József Pelikán (shared 2nd prize, 1965), Imre Ruzsa (shared 2nd prize, 1970, and shared 1st prize, 1971). Source: [32, pp. 13-14].

1929. At 16, Erdős studies Knopp's "Lehrbuch der Functionentheorie" with *Apuka*.

1930. At 17, Erdős enrolls in the mathematics program at Péter Pázmány University[56], Budapest. He meets Turán, three years his senior. His first question to Turán is the divergence of $\sum 1/p$, a question *Apuka* has been unable to answer.

1930-34. At the university Erdős meets many of the colleagues he has known through KöMaL. He joins the weekly excursions of a small group of mostly Jewish young mathematicians, whose number includes his closest friend Tibor Gallai, as well as Turán, Márta Wachsberger (now Martha Sved), Eszter (Esther) Klein, Endre Vázsonyi, László Alpár, Géza Grünwald, Dezső Lázár, and a remarkable student of chemical engineering by the name of György (George) Szekeres.

Erdős's major mathematical influences through the university include some of the members of this group as well as the lectures by Professors Leopold Fejér and Pál Veres at Pázmány University, Mihály Bauer, József Kürschák, Gusztáv Rados, and Dénes König at the Technical University across the Danube river, and László Kalmár in Szeged. Erdős contributes an infinite version of Menger's theorem to König's classic book, the first monograph on graph theory, published in 1936.

1930. Erdős gives a simple proof of Chebyshev's theorem that there is a prime between n and $2n$. László Kalmár, a versatile mathematician from Szeged working mainly in mathematical logic and the theories of real and complex functions, helps him polish his first publication, which appears in *Acta Scientiarum Mathematicarum, Szeged*, in 1932.

1931. The Erdős family moves to a spacious new apartment at 8 Abonyi Street, across from the Jewish high school where later Gallai will teach.

1931. Concluding his studies in Göttingen and Berlin, Richard Rado completes his dissertation entitled "Studien in Kombinatorik" under Issai Schur. A "mathematical jewel" [28], the dissertation gives a common generalization of Schur's and Van der Waerden's theorems on monochromatic patterns of integers. This is an area closely related to Ramsey theory, which will become a major area of common interest of Rado and Erdős.

[56]The university was renamed Loránd Eötvös University in 1948.

1932. During his second undergraduate year, Erdős essentially completes his Ph. D. thesis under Leopold Fejér. The main results: simple proofs of the existence of primes between n and $2n$ for certain arithmetic progressions such as $4k-1$ and $8k+1$. Erdős has to wait two more years before he can graduate and submit the thesis. The results appear in the *Mathematische Zeitschrift* (1935).

1932. László Alpár, Erdős's friend from the Anonymous group, wins the Eötvös competition. Soon thereafter he is jailed for antifascist activities and membership in the communist party. He is released after a month.

1932. Motivated by a problem in plane geometry initiated by Eszter Klein (the "Happy End Problem"[57] [cf. 1936]), Szekeres rediscovers Ramsey's theorem which becomes one of the lifelong obsessions of Erdős. This obsession is contagious (see [34]) and results in *Ramsey theory*, both finite and transfinite, the latter being called *partition calculus*, after a monumental Erdős-Rado paper (1956). These theories play a fundamental role in combinatorics and in set theory.

1933. January 30: Hitler becomes Chancellor of the German Reich. Persecution of German Jews begins immediately. Mass exodus of German-Jewish scholars follows. Richard Rado receives his doctorate in Berlin but as a Jew, he is denied access to academic employment and is even expelled from the German Mathematical Association. His life is saved by a scholarship to Cambridge. Later he is able to help his mentor, Issai Schur, to emigrate.

1933. Erdős reveals his elementary proof of Schur's conjecture on abundant numbers to Turán the last day of the year. (A proof via Fourier analysis was previously found by Davenport, Chowla, and Behrend.) The proof leads him to the problem of the distribution of values of real-valued additive functions, investigated later in the Erdős–Wintner and Erdős–Kac papers (1939–40). Issai Schur calls 20-year-old Erdős "the wizard from Budapest" and includes the proof in his lectures.

1934. At 21, Erdős graduates from Pázmány University with a Ph. D. The degree at this age is highly unusual.

[57] See the section on "Anonymous."

1934. September: away from home for the first time. Erdős holds fellowships at the University of Manchester (arranged by Louis Joel Mordell) and the Royal Society. His remuneration: £100 for 6 months. On his way to Britain, Erdős stops in Zürich and meets George Pólya.

Erdős tries his powers at putting butter on his bread for the first time at tea at the Davenports' in Cambridge. (He gets encouragement and succeeds.)

1934-38. Erdős in Manchester. He visits home three times a year (Christmas, Easter, summer).

1934. Autumn: Erdős meets Rado in Cambridge; a legendary collaboration, spanning more than half a century, begins.

1935. Erdős meets Stanislaw Ulam in Cambridge.

1936. The Happy End: George Szekeres and Esther Klein get married. "I remember the wedding day," Erdős says, "it was just a day after I learned that Vinogradov had proved the odd Goldbach conjecture."

1936. László Alpár emigrates to France and becomes active in the antifascist movements there.

1938. In a far-sighted anticipation of *extremal set theory*, the Erdős–Ko–Rado theorem is born (cf. [12]). Upon publication more than two decades later (*Quarterly Journal of Mathematics*, 1961), the paper rapidly becomes a classic (cf. [7]).

1938. March 12: *Anschluss*, the annexation of Austria into the Third Reich. September 29-30: Munich pact, dismemberment of Czechoslovakia with Western consent.

1938. September 3: Alarmed by Hitler's demands on Czechoslovakia, Erdős leaves Hungary in a great haste. He travels through Italy, meets with George Pólya in Zürich and with László Alpár in Paris. After three weeks in Britain, he leaves for the U. S. on September 28 and takes up his fellowship at the Institute for Advanced Study, with a stipend of $1500 for one year.

1938-48. Erdős spends the decade in the U. S.

1939. Summer: Erdős concludes what even in 1995, after 1400 articles, he considers his most successful academic year. The crop includes: the Erdős–Kac and Erdős–Wintner theorems on additive functions, his third paper with Turán on interpolation (*Annals of Mathematics*, 1940; the preceding papers in the series also appeared in the respected Princeton periodical), and the result that the product of consecutive integers is never a square.

To his dismay, his IAS fellowship is not extended and he is left penniless. "Leopold Infeld wasn't extended either," he comments. But he also remarks, with more than a little bitterness, that Von Neumann did not lift a finger to help him.

He is living on fifty-dollar loans from colleagues. He is unable to send money to his favorite aunt in Slovakia, whom some bribes and a trip might have saved from the Nazis.

Once Wintner publicizes Erdős's situation, Erdős receives another half year's stipend.

1939. In September at the Institute in Princeton, Erdős meets Witold Hurewicz. As usual, Erdős asks him for a problem. Hurewicz mentiones the then outstanding problem in *dimension theory:* determining the topological dimension of the set of rational points in separable Hilbert space. The experts' guess is that it must be either zero or infinity. Erdős needs one more piece of information, the definition of dimension. He then proves on the spot that the dimension is positive and a few days later he shows that the correct value is 1 (*Annals of Mathematics*, 1940).

1939. September 1. Hitler attacks Poland. World War II begins. On September 5, the U. S. proclaims its neutrality.

1939. When the war begins, Alpár is promptly arrested by the French and put in an "internment camp" for his antifascist activities. He will escape in 1943 and join the French *Résistance*.

1940-43. Erdős gets a stipend from the University of Pennsylvania (Philadelphia).

1941. June 22. Hitler attacks the Soviet Union.

1941. Hungary joins Germany in invading the Soviet Union. Postal correspondence between the U. S. and Hungary ceases. For four years, Erdős will only hear sporadic news about his parents and friends.

1941. Stanislaw Ulam invites Erdős to give a colloquium lecture at Madison, Wisconsin. In his *"Adventures of a Mathematician,"* Ulam writes [44, p. 134]:

> "In 1941 he was twenty-seven years old, homesick, unhappy, and constantly worried about the fate of his mother who had remained in Hungary.
>
> "At that time he was even more in perpetual motion than now – almost constantly jumping up and down or flapping his arms. ... If some amusing thought occurred to him, he would jump up, flap his hands, and sit down again.
>
> "During this visit and a subsequent one we did an enormous amount of work together."

Erdős and Ulam work on set theory. Although they announce some of their results at an A. M. S. meeting, no joint papers of the two emerge until much later (1968).

Some of their unpublished early results are later rediscovered by an Indian mathematician (Erdős could not recall his name). When the mathematician informs Erdős of the result, Erdős suggests that he should publish it. Only several years later does he find out that Erdős had the result earlier. "Why didn't you tell me?" "I don't want to imitate Gauss[58] in this regard."

1941. August: *Erdős arrested on Long Island as a spy.*

After a visit to German mathematician Gerhard Hochschild at Washington Heights, Art Stone takes Erdős and Shizuo Kakutani on a tour of Long Island. They retire to a motel; Erdős notices an interesting source of light some distance away which they decide to check out the next morning. Stone, then a driver with little experience, overlooks the "No trespassing" sign near the light. Kakutani takes pictures of Erdős and Stone; the background turns out to be a radar installation. The guard warns them, so they drive off. They stop for lunch at the far end of Long Island. After the meal, two "enormous" policemen arrest them. It turns out that the guard has reported that "three Japanese took pictures of the installation and then departed in

[58] Gauss used to tell young mathematicians presenting him brilliant new results that he could not praise the results since he would be praising his own unpublished work of decades earlier. Jacobi responded defiantly; János Bolyai's career, and life, was shattered.

a suspicious hurry." FBI agents are on their way to interrogate them. They are being questioned separately. By the evening, Kakutani is cleared of the suspicion of being a "Japanese agent;" they are released amidst apologies. But they do share the blame, they are told, how could they miss the "No trespassing" sign. "I was thinking," Erdős explains. "What about?" "Mathematics."

His record thoroughly checked, the incident will save Kakutani some trouble later.

1942. Erdős's paper "On the law of the iterated logarithm" appears in the *Annals of Mathematics*. It deepens the theory of random fluctuations of coin-tossing sequences and of Brownian motion.

1942. In a paper that appears in the *Revista de la Univ. Nac. de Tucuman*, Erdős lays the foundations of *transfinite Ramsey theory*, one of the great theories originated by Erdős. (Cf. [23].)

1942. At Notre Dame, Erdős learns from a newspaper that his friend, brilliant young mathematician Géza Grünwald, has been murdered by a guard in a "military labor company" operating on the Soviet front. The same fate befalls Henrik Popper, husband of Erdős's aunt Irma.

1942. *Apuka*, Erdős's father, suddenly dies of heart attack in Budapest. Erdős receives the news from László Alpár who manages to write him from the internment camp in Paris. *Anyuka* goes through a period of denial and deep depression. Erdős unsuccessfully tries to send messages to Budapest through various channels, including friends in South America.

1942. Tormented by anguish akin to that of Erdős, Kakutani returns to Japan on a Swedish boat to tend his mother. His paper on the decomposition of the reals into countably many Hamel bases (under CH) is rejected by the *Duke Mathematical Journal* on the grounds that "the author is Japanese and did not show his disagreement with the Japanese government"[59]. The paper appears in the *Transactions of the American Mathematical Society* in 1943. – Kakutani's mother dies of natural causes during the war. Kakutani survives and returns to become a leading figure on the mathematical scene of postwar U. S.

[59] Recollection by Erdős.

1943. The joint paper of Erdős with Tarski "On families of mutually exclusive sets" appears in the *Annals of Mathematics*. This paper includes the first theorem about *inaccessible cardinals*. Inaccessibility arises as the surprising equivalent of the existence of set systems with a natural combinatorial property. Perhaps the most important parts of the paper are its footnotes, listing, without proof, combinatorial properties of inaccessible cardinals, marking the beginning of *the theory of large cardinals*. This theory will become a fundamental part of set theory. (Cf. [23].)

1943. Erdős hears from *Anyuka* in a brief message forwarded by the Red Cross.

1943-45. Finally, a real job! Erdős is a "research instructor" at Purdue University. Ulam recalls that in this period Erdős is no longer penniless, "even out of debt" [44, p. 135]. His hosts are Art and Dorothy Stone. He works on number theory with Leon Alaouglu and Ivan Niven.

1944. *Erdős thinking, again.* Erdős takes a midnight walk near Purdue University. He looks suspicious to the Crime Prevention Team of the University police. They stop him and ask what he is doing. "I am thinking." "You are," the reply goes and the Team drives on.

1944. Ulam invites Erdős to participate in the nuclear project and advises him to write to Edward [Teller]. Erdős is interested but writes to "Professor Edwards" that after the war he might want to return to Hungary. This desire disqualifies him.

1944. March 19: Hungary under German occupation. Systematic annihilation of Hungarian Jewry begins. The losses among Erdős's relatives are beyond comprehension.

1945. February 13. After four devastating months of *Nyilas* terror, Budapest is liberated by the advancing Soviet troops.

Erdős's mother is now readmitted to public teaching but chooses to be an editor with the Hungarian Academy of Science instead.

Gallai teaches at Abonyi Street Jewish High School. After classes he often visits Erdős's mother who lives across the street with her sister *Irma néni*[60] and Irma néni's daughter. Vera Sós is among Gallai's students.

[60] *Irma néni:* Aunt Irma.

1944-45. Erdős receives a Guggenheim Scholarship and spends it mostly at Stanford. In correspondence from Ann Arbor, the Erdős-Stone theorem, a cornerstone in extremal graph theory, is completed (*Bulletin of the A. M. S.*, 1946).

1945-48. Erdős is affiliated with Syracuse University.

1945. *Turán's ID.*

Under orders from Stalin to fill quotas of prisoners to be sent to the *GULAG*, the majority never to return, the liberators pick men in the streets at random for the *malenkiĭ robot*[61].

Fresh from the ordeal of "labor companies" and *Nyilas* terror, Turán is stopped by a Soviet patrol in Budapest. The officer demands his papers. Having escaped from a *Nyilas* round-up a few days earlier, Turán has no proper ID. He does carry, however, a reprint of his 1935 paper with Erdős, published in the *Bulletin of the Institute of Mathematics and Mechanics of Tomsk*, the institute where number theorist Romanov has worked. The officer is impressed by the prewar publication in a Soviet journal and lets Turán go. "An unexpected application of number theory," Turán reports to Erdős.

1945. August. Erdős receives a telegram from a friend in Arad, Romania. He is overjoyed to learn that *Anyuka* is alive. Erdős is at Stanford in December when he receives the first postwar letter from *Anyuka*.

1946. *Erdős invited to the Ulams.*

In a hospital near Los Angeles, Stanislaw Ulam is recovering from a severe ailment tentatively diagnosed as viral encephalitis. First, he and his friends fear for his life, then he fears his mental abilities would remain impaired. In his *"Adventures of a Mathematician"* [44, pp. 178-179], Ulam recalls:

> "I remember being discharged from the hospital. As I was preparing to leave, fully dressed for the first time, standing in the corridor with Françoise [Ulam's wife], Erdős appeared in the end of the hall. He did not expect to see me up, and he exclaimed: 'Stan, I am so glad to see you alive. I thought

[61] Small labor.

> you were going to die and that I would have to write your obituary and our joint papers.' ...
>
> "Erdős had a suitcase with him and was just leaving after a visit to Southern California. He had no immediate commitments ahead and said, 'You are going home? Good, I can go with you.' " [So he went, and stayed ...] "No sooner had we arrived than Erdős proposed a game of chess."

Ulam, not sure at first whether he could remember the rules, wins twice in a row, whereupon Erdős declares he is tired. Ulam sees that Erdős has played in earnest. For nearly two weeks, Erdős would continue to bombard his host with mental challenges. This "therapy" helps Ulam regain his self-confidence.

1948. Turán arrives in Princeton in January and spends a half year at the Institute. He does a lot of work with Erdős on number theory, distribution of roots of polynomials, and other topics.

1948. Erdős meets Atle Selberg at the Institute for Advanced Study in July; an elementary proof of the prime number theorem emerges.

1948. Erdős obtains his "green card" (permanent residence permit in the U. S.). Soon afterwards he embarks on a trip to Europe, leaving the U. S. for the first time in a decade.

1948. Autumn: Erdős spends two months in the Netherlands. He works with De Bruijn on combinatorics, number theory, and analysis, and with Koksma on number theory.

He also meets childhood acquaintance Alfréd Rényi. At the university, Erdős's parents had taken aesthetics classes from Bernát Alexander, grandfather of Rényi, and became friends of the family.

Eight years Erdős's junior, Rényi emerges as a mathematical genius working in a great variety of fields, including number theory, probability theory, orthogonal series, information theory, combinatorics, and applied mathematics. He also writes literary essays on mathematics (cf. Turán [40]).

His entry to the university delayed by the Numerus Clausus, Rényi becomes a resistance hero during the *Nyilas* terror in Budapest. Hav-

ing escaped from a death march, he dresses in a *Nyilas* uniform and saves others at grave risk to his own life[62].

He completes his graduate studies in Leningrad under Yuri V. Linnik in less than two years (1947), producing a new sieve method in number theory and proving one of the strongest results in existence toward the Goldbach conjecture.

During his short but stellar career, he becomes one of the most influential collaborators of Erdős. They obtain their first results together in Amsterdam, "On consecutive primes."

1948. For the first time in a decade, Erdős visits Budapest. He arrives in his native town on December 2 and spends two months with his mother and friends who survived the Holocaust. Gallai introduces brilliant undergraduate Vera Sós to him. Erdős visits a kindergarten class, perhaps to see Gallai's little niece Éva Bogdán, and finds in the same group a 5-year-old boy by the name of Miklós Simonovits, son of his friend Anna Beke, former member of the *Anonymous* group. Sós and Simonovits later become major collaborators of Erdős's and of each other, mainly in extremal combinatorics and Ramsey–Turán problems. After the death of *Anyuka* (1971), they, along with András Hajnal, will be Erdős's most intimate, familiar friends.

Erdős resumes his work with Turán on the distribution of roots of polynomials (*Annals of Mathematics*, 1950).

1949. February: Erdős visits a number of places in the U. K.

March: Erdős returns to the U. S. and spends the spring term at Syracuse University. During autumn, 1949 and winter, 1950 he is again without a job.

1949. Following the model of Stalin's purges, scripted trials are arranged in Hungary by "Big Brother" Mátyás Rákosi, the communist dictator of the country. Fellow communist László Rajk is convicted on fabricated charges of espionage on behalf of Yugoslavia, and is executed. A seasoned political prisoner, László Alpár is arrested for his former association with Rajk, and sentenced to forced labor in a copper mine. He serves four harsh years before the thaw in politics following Stalin's death (1953) returns his freedom.

[62] Erdős's account.

1950-51. Erdős is in Aberdeen, U. K.

1950. The International Congress of Mathematicians is held in Cambridge, Massachusetts. "Mathematicians from behind the Iron Curtain were uniformly prevented from attending the Congress by their own governments," conference secretary J. R. Kline reports. Atle Selberg shares the Fields Medal with Laurent Schwartz.

1950. Mathematicians from the communist countries hold their own congress in Budapest. Hua is among the Chinese delegates. Led by the large Soviet delegation, the participants pay warm tribute to 70-year-old Fejér and F. Riesz. Erdős is not invited to the meeting because his colleagues fear he would not be able to leave the country.

1950. Following the Soviet model, the Mathematical Institute of the Hungarian Academy of Science is established. Directed by Alfréd Rényi from its inception until his untimely death in 1970, the Institute develops into a world-class center of scholarship and continues to thrive under subsequent directors László Fejes-Tóth, András Hajnal, and Domokos Szász. It maintains an atmosphere of intellectual freedom unaffected by the dogma of the era.

This atmosphere suits Erdős well. The Institute will serve as his home base for decades to come. It is the setting where he will meet generations of new *epsilons*, and the birthplace of countless joint papers.

1951-52. Erdős works at University College, London, with Davenport.

1951. *Cole Prize.* On December 28, at the business meeting of the Annual Meeting of the American Mathematical Society, held at Brown University, A. M. S. President John von Neumann announces that the Frank Nelson Cole Prize in Number Theory was awarded "to Paul Erdős, for his many papers in the Theory of Numbers, and in particular for his paper *On a new method in elementary number theory which leads to an elementary proof of the prime number theorem*, Proc. Nat. Acad. vol. 35, pp. 374–385, July 1949, in which he makes important contributions to the elementary theory of primes inaugurated by A. Selberg."[63]

At the same meeting, Shizuo Kakutani gives an invited address on "Brownian Motion," subject of his then current joint work with Erdős and Anton Dvoretzky.

[63] Citation from the *Bulletin of the A. M. S.,* vol. 58 (1952), pp. 159–160.

1952. Erdős finally lands a secure job at Notre Dame University. It would last several years and involves little teaching.

1953. A sad note on how easily citizens of a free country submit to fear. *Anyuka* has her 73rd birthday on July 6 in Budapest. Erdős, who has not seen her for five years, wants to call her from a friend's home in Los Angeles. The friend refuses, out of fear of becoming associated with a phone call to a communist country.

1953. In October at an A. M. S. meeting at Columbia University, Kakutani presides over a lecture by Erdős on "Combinatorial Problems in Set Theory," another area in which they had collaborated. The speaker is listed as "Professor Paul Erdős of Notre Dame University."

1954. *McCarthyism catches up with Erdős.* Erdős is interviewed by the Immigration Service and his reentry permit is denied. Erdős chooses to attend the International Congress of Mathematicians in Amsterdam without the permit. He cannot return to the States for nine years (except for a brief visit on a "special visa"). He loses his "green card" and his Notre Dame job.

1954. Erdős spends three months at The Hebrew University, Jerusalem, and gives one lecture per week.

Ernst Specker earns $20 by solving a prize-problem of Erdős in set theory. He is the first one to claim a monetary reward from Erdős.

1954. Turán visits the USSR and China. He takes the Trans-Siberian train from Moscow to Beijing, an eight day trip, which he enjoys.

1955. Erdős spends three months at the Technion, Haifa. He works with Hanani on combinatorics and with Jabotinski on analytic number theory and the theory of functions. Erdős becomes a "permanent visiting professor" there. His friend Haim Hanani takes him to a high school with a program for mathematically gifted children. For decades, Erdős will return to give an occasional lecture. Ron Aharoni is among those young collaborators of Erdős whom he has first met in these classes.

1955. August: Erdős visits Hungary for the first time since 1948. The country tightly locked behind the Iron Curtain, foreign travel is virtually impossible, correspondence restricted and letters are opened by the

authorities. Yet, through a friend with high connections, Erdős, then and now a Hungarian citizen, is assured of free movement.

During his visit, Erdős meets set theorist Géza Fodor of the University of Szeged, who becomes his first Hungarian collaborator in set theory.

1955. Autumn: Erdős works with Karl Prachar in Vienna on number theory and attends a number theory conference in Belgium.

1956. Erdős is elected to membership in the Hungarian Academy of Science. This would have been unthinkable during the years of strict Stalinism (1949-53).

1956. On a visit to Szeged, Erdős meets Fodor's colleague András Hajnal, a graduate student of logician László Kalmár. Within a day, the foundations of a joint paper are laid, the beginning of a long and intense collaboration.

As Hajnal's office turns into Erdős's home base, their joint work of more than 50, often voluminous, papers shapes *transfinite combinatorics*.

1956. October–November: The Hungarian revolution against Communism and Soviet domination is crushed within two weeks by Soviet tanks. Installed by the Soviets as head of government and Party, János Kádár remains in power for 32 years. Over the decades, his regime becomes less oppressive and eventually the most "liberal" in the Soviet bloc.

1957. Erdős visits Hungary in July, after a renewed promise of free movement by the new government. He spends several weeks with *Anyuka* at the resort of the Hungarian Academy of Science in Mátraháza among the woods and hills of the Mátra mountains. For more than a decade, they would return to Mátraháza two or three times a year. They are never alone; the presence of Erdős turns the resort into a mathematician's Mecca. Turán and his wife Vera Sós, Alfréd and Catherine Rényi, András Hajnal, and number theorist Péter Szüsz are their most frequent company. Erdős, as usual, collaborates simultaneously with all of them. Their entertainment includes hiking, table tennis, chess, chess puzzles, and helping *Anyuka* with her cross-word puzzles.

The visits of Erdős to Mátraháza become less frequent after the Rényis and *Anyuka* pass away between 1969 and '71, and Turán in '76. Hajnal,

the remaining main partner, prefers the lakeside, so Erdős becomes a more frequent guest at the Academy's holiday resorts at Lake Balaton.

1958. After spending the winter in Vancouver, Erdős returns to Hungary to receive the *Kossuth Prize* from the government on April 4.

Erdős is still in Hungary when, on June 16, Imre Nagy, head of the revolutionary government of 1956, is executed against Kádár's previous promises to the contrary. A couple of months later at the International Congress of Mathematicians in Edinburgh, Erdős discusses with Alfréd and Catherine Rényi his wish to protest the execution by not returning to Hungary. Eventually, out of consideration for his mother, he drops the idea and visits Budapest together with George and Esther Szekeres, who have not been back there for twenty years.

The Edinburgh Congress hosts a large delegation from Hungary (about 25 mathematicians), a sign of new times in spite of Nagy's murder. The regime's new-found courtesy is somewhat limited, though; the delegates arrive without a penny in Western currency. Erdős bails them out. Hajnal receives a loan of £25 and takes years to repay; some friends receive the sum as a gift. £25 is a substantial sum in 1958; their hotel costs £1 per night.

At Edinburgh, Erdős meets Eric Milner, a former student of Rado, who is working in Singapore. Milner will become one of Erdős's major collaborators in set theory.

1958. Erdős takes a boat from Athens to Haifa. With no mathematical company at hand, he embarks on a systematic study of *extremal problems in graph theory*. He reports his results much later, at the 1963 conference in Smolenice, Slovakia (proceedings published in 1964).

1958. Erdős obtains a "special visa" to attend an A. M. S. meeting in Boulder, Colorado.

1959. Erdős meets 12-year-old child prodigy Lajos Pósa. Erdős keeps in touch with him through correspondence and phone calls. By the age of 16, Pósa has published a series of influential papers in extremal graph theory.

1959. Erdős attends the International Conference on Graph Theory, the first one ever on the subject, held at a modestly equipped tourist hostel at Dobogókő, a scenic hilltop in Hungary's Transdanubian mountains.

Many of the best Hungarian mathematicians are present, regardless of their main field of interest, including Gallai, Turán, Sós, Rényi, Hajnal, algebraist George Grätzer; a good opportunity for Erdős to work simultaneously on number theory, interpolation, graph theory, set theory, and other subjects, with suitable partners.

1960. On a visit to Szeged, Erdős meets another child prodigy, 13-year-old Attila Máté. An eager problem solver, Máté takes on set theory. Later he will turn to classical analysis.

1960. Erdős travels to the USSR and China. His itinerary: Budapest, Moscow, Leningrad [St. Petersburg], Moscow, then the long flight to Beijing with a night's rest in Irkutsk (Siberia) and another stop in Ulan Bator (Mongolia). He spends 3 weeks in China, meets with his old friends Hua and Chao-Ko. From Beijing he flies to Shanghai, takes the train to Hangchow, then flies to Canton. From Canton he takes the train to Hong Kong and then flies to Singapore to work with Milner. His subsequent destination is Australia.

1962. Erdős is officially affiliated with the Mathematical Institute of the Hungarian Academy of Science. He receives (modest) pay while in Hungary.

1963. At the U. S. Consulate in Vancouver, Erdős is told that "the State Department determined that you are a Communist." Finally, the U. S. government reviews the file on Erdős and drops allegations that he belonged to "proscribed organizations." Erdős receives an "exchange visitor" visa (J-1) which he has to renew periodically.

1963. A recent Berkeley graduate employed by Bell Laboratories, Ronald L. Graham meets Erdős for the first time at a number theory meeting in Boulder, Colorado. His 5th paper (1964) bears the title "On a conjecture of Erdős in additive number theory," not a very informative title as it turns out, given the number of problems of Erdős in the area. Erdős and Graham begin working on their first joint paper in the late 60s. Graham attends virtually all international combinatorics conferences held in Hungary, beginning with his first visit in 1969 for the Balatonfüred meeting. In the early 70s, Erdős becomes a regular visitor at Bell Labs in New Jersey, and a house-guest at the Grahams, typically 3–5 days at a time. Graham extends his care to many earthly aspects of Erdős's life (including handling his finances).

Erdős and Graham do seminal joint work in *Euclidean Ramsey Theory*. Their collaboration in number theory leads to their 1980 monograph [13]. A followup as a Springer volume is now in preparation.

1964. *Anyuka* relinquishes the Abonyi Street apartment, home to three decades of fond as well as tragic memories. She trades the family's home for two smaller apartments. Together with her sister, *Irma néni*, she moves into an apartment on Németvölgyi street, The building is located in a nice part of Budapest, at the foot of the hills of Buda, next door to the Turáns and other members of the Hungarian Academy of Science. The move finally enables Irma néni's daughter to live in a separate apartment.

Like most apartments in this period of "socialist" government, both the old and the new Erdős residences are state owned. However, the low rents make tenancy practically a form of ownership, recognized by cash payments when apartments of unequal value are traded. Only near the end of the communist era will the government offer the apartments for sale. Erdős will purchase the Németvölgyi út apartment in 1988.

1964. At 84, *Anyuka* joins her son on his never-ending journey around the world. They visit Israel in November, and Britain the next year; the U. S. and Canada a year later. She vigorously sets out to improve her English. They become inseparable until her death seven years later. Contrary to news reports, she is comfortable with traveling. Her often misinterpreted statement is this: "I do not travel because I like it but because I want to be with my son." She disproves her own prophesy that "children become letters."

1965. Vera Sós and András Hajnal launch their combinatorics seminar at the Mathematical Institute of the Hungarian Academy of Science.

1965. The "Giant Triple Paper" (over 100 densely packed pages) by Erdős, Hajnal, and Rado on partition calculus appears in the *Acta Mathematica Hungarica*. Lower bounds on transfinite Ramsey numbers follow from their "negative stepping up lemma."

1967. The Milners move to Calgary. Erdős becomes a frequent visitor.

1968. Erdős meets 15-year-old Imre Ruzsa. Within a year, Ruzsa is well on his way to becoming a star in combinatorial number theory.

1969. In Los Angeles, Joel Spencer presents Erdős with a solution to the latter's problem on ranking tournaments. A long collaboration on the *probabilistic method*, one of Erdős's trademark techniques, follows. The first monograph on the subject, by Erdős and Spencer, appears in 1974. Spencer edits a volume of selected papers of Erdős (1973). He spends the academic year 1975–76 in Budapest with his family, and enters into collaboration with a number of Hungarian combinatorists. A master expositor, he becomes a vocal promoter of what he calls his "Hungarian mathematics." He is one of the founders of the journal *"Random Structures and Algorithms,"* explicitly linking a field created by Erdős and Rényi to the *theory of computing.*

1969. Catherine Rényi, complex analyst, dies tragically at a young age shortly before the beginning of an international combinatorics conference in Balatonfüred. Her husband Alfréd Rényi addresses a plenary session of the meeting, reporting their joint work, still in progress a few days before, on a theory of trees.

1969. Erdős and *Anyuka* spend three months together in Canada, a week in Hawaii, and three months in Australia. At a combinatorics conference in Calgary, Vera Sós introduces the *Ramsey–Turán-type* problems, one of the central subjects of her future collaboration with Erdős.

1970. February 1: Alfréd Rényi dies of cancer at the age of 49. Unaware of his condition[64] and feeling "healthy," he has left the hospital in mid-January against his doctor's advice and has announced his plans for a course on a theory of trees to eager students at Eötvös University.

1970. Erdős is invited to speak at the first Louisiana Conference on Combinatorics, Graph Theory, and Computing. He is taken ill and cannot speak. The conference, renamed Southeastern Conference on CGTC in 1972, becomes an annual event, roughly alternating between Baton Rouge, Louisiana, and Boca Raton, Florida. The highlight each year is the Thursday afternoon lecture by Paul Erdős on "Problems and results in combinatorics," with an audience averaging about 350.

[64] In Hungary, doctors used to go to great length to conceal a terminal condition from the patient. This must have required particularly skillful deception in the case of Rényi, a person of great erudition, whose interests included medicine. Even to date, doctors prefer to tell the truth to an immediate relative, and defer to the relative's judgment as to how much to reveal to the patient.

Erdős missed only one meeting since 1972. On one occasion, organizer Fred Hoffman had to declare Friday to be Thursday to preserve the tradition.

1970-71. *Anyuka* complains of abdominal pain and is rushed to a hospital in Calgary. She dies of peritonitis, after brief suffering.

Erdős gets amphetamines as anti-depressants. Later he uses them as mental stimulants to fuel his 19-hour workdays. He begins referring to himself as "P. G. O. M."[65]

1972. Erdős's aunt *Irma néni* dies.

1973. *Faudree converted.* Group theorist Ralph Faudree meets Paul Erdős and abandons group theory. Their long friendship and productive collaboration on *generalized Ramsey theory* begins.

From 1974 on, Erdős becomes a frequent visitor at Memphis and acquires an official position as "adjunct professor." He works with Richard Schelp, Cecil Rousseau, and especially with Faudree. Faudree spends two academic years with his family in Budapest. His children "grow up with Uncle Paul."

1973. *Erdős's ID.* Erdős walks across a red light near UCLA at night, causing a turning police car to stop. "Can I see your ID, sir?" As usual, Erdős has no documents on him. All he carries is a thick volume, *"The Art of Counting,"* fresh out of press. That volume, with Erdős's 8×5 photo smiling at the officers, is readily accepted as proof of his identity for the purposes of the $10 ticket he is subsequently served for jay-walking. Bruce Rothschild pays the fine.

1973. "International Conference on Infinite and Finite Sets" is organized by the Bolyai Society in Keszthely, Hungary, to honor Erdős's 60th birthday. Friends and collaborators from around the globe gather. The meeting produces a proceedings of three huge volumes. Volume 2 consists entirely of papers written by Erdős with coauthors (including two papers with Graham, Montgomery, Rothschild, Spencer, and Straus, laying the foundations of *Euclidean Ramsey Theory*).

The Hungarian government denies entry visas to Israeli mathematicians. Erdős is deeply offended. He considers boycotting the meeting

[65] "Poor Great Old Man"; see the section on "Linguistics etc."

itself but backs down on his friends' urging. He leaves the country after the meeting and does not return for several years out of protest.

1973. Autumn: Erdős's destinations after Keszthely are Israel, the U. S., England, and Australia in quick succession.

1974. Endre Szemerédi solves a $1000-dollar problem of Erdős. The problem was originally proposed by Erdős and Turán in 1936.

1975. Erdős and Turán meet in Canada.

1976. September 26: lifelong friend and collaborator Paul Turán dies of leukemia. The news of his terminal illness brings Erdős back to Budapest, ending Erdős's self-exile.

Ever since that year, Erdős has returned to Hungary frequently, building intense collaborations with new generations of mathematicians. Unwilling to return to the home he used to share with *Anyuka*, he stays in the guest-house of the Hungarian Academy of Science. He has not given up the apartment, though. He has offered it to visitors, always keeping track of who comes and when. More recently, he invited a young mathematician to move in with his family, free of charge, while he continued to stay away.

Even this silent vow did not last forever, though. I first interviewed *Pali bácsi* for this article in December 1994; to my surprise, the interview took place in the same old apartment where, nearly three decades earlier, I had the honor, at the age of 16, to be treated by *Pali bácsi* to a (well-ordered) set of problems on ordinal numbers. *Anyuka* was looking on intently, occasionally interjecting the question, "Pali, does he understand this?"

Let me stop on this personal note, and leave the story of Erdős's amazingly productive recent decades to another time and, *especially*, another person.

APPENDIX

The lists given here are largely based on Paul Erdős's recollection but most data have been verified and completed using independent sources. Errors and omissions are nevertheless likely. I will be grateful for corrections and additions.

Current employments

Paul Erdős has a permanent affiliation with the Mathematical Institute of the Hungarian Academy of Science and receives a year-round salary.

In addition he holds several part-time positions in the U. S. and one in Israel. Typically he receives pay while present.

Technion, Haifa (Erdős typically spends 2 months per year there)

University of Florida, Gainesville (two weeks per year)

University of Memphis (formerly Memphis State University) adjunct professor informally since 1975 and officially since 1977; paid for one week per year (but Erdős visits on average three times and spends an average total of one month there each year without extra pay)

Western Michigan University, Kalamazoo, Michigan: adjunct professor since September 1989. Lately he has been visiting two or three times a year, spending a total of 10-14 days there. He underwent a major medical procedure in Kalamazoo in autumn, 1989 and the hospital waived all fees. On each visit he receives courtesy medical checkups, arranged by his old friend, graph theorist Yousef Alavi.

Prizes and awards founded by Paul Erdős

Mathematical Prize (Hungarian Academy of Science, 1972)

Endowed by Paul Erdős to commemorate his parents, the prize is awarded annually to young mathematicians in Hungary. The prize is administered by the Hungarian Academy of Science. The first recipient was István Juhász in 1974, followed by Gábor Halász (75), Endre Szemerédi (76), László Lovász (77), Ferenc Schipp (78), Zoltán Daróczy (79), Gábor Tusnády (80), András Sárközy (81), László Babai (Feb. 83), Ferenc Móricz (Dec. 83), József Beck (84), János Pintz (85), Sándor Csörgő (86), Miklós Laczkovich (87), Imre Ruzsa (88), Péter Komjáth (89), Ágnes Szendrei (91), Antal Balog (92), Péter Pál Pálfy (93), Bálint Tóth (94), Imre Bárány (95).

Erdős Prize (Israel Mathematical Union, 1976)

Endowed by Paul Erdős to commemorate his parents *Anna* and *Lajos Erdős,* the prize is awarded to young mathematicians in Israel by the Israel Mathematical Union. The award is made biennially except when the IMU finds that the interests accrued suffice for

awards in consecutive years. The first recipient was Saharon Shelah in 1977, followed by Elia Rips (79), Ofer Gaber (81), Adi Shamir (83), Shmuel Kiro (85), Yosef Yomdin (87), Noga Alon (89), Alex Lubotzky (90), Gil Kalai (92), Ehud Hrushovski (94).

The Paul Turán lectureship (János Bolyai Mathematical Society, 1977)

Endowed by Paul Erdős and Vera Sós to commemorate Paul Turán (1910-76), the lectureship supports one-week visits by distinguished speakers in areas encompassed by Turán's work. The foundation is managed by the Bolyai Society; most lecture series are held at the Mathematical Institute of Hungarian Academy of Science (Budapest).

The speakers so far have been Alan Baker (1978), Klaus F. Roth (1979), Lennart Carleson (1981), Wolfgang M. Schmidt (1984), Andrzej Schinzel (1984), Jean-Pierre Kahane (1984), Yakov G. Sinaĭ (1985), Atle Selberg (1985), Enrico Bombieri (1987), Grigoriĭ A. Margulis (1989), Richard A. Askey (1992), Robert Tijdeman (1994), Henryk Iwaniec (1995).

The Anna Erdős postdoctoral fellowship (Technion, Haifa, 1984)

Paul Erdős used more than half of his Wolf Prize award to endow a postdoctoral fellowship in mathematics at the Technion to commemorate his mother. The fellowship is offered about once every five years. The first recipient of the fellowship was Joseph Horvath, 1990-91. The second fellowship will probably be offered for 1996-97.

Prizes and awards received by Paul Erdős

Cole Prize, A. M. S. 1951 (for his 1949 paper in the *Proceedings of the National Academy of Science* "On a new method in elementary number theory which leads to an elementary proof of the Prime Number Theorem")

Kossuth Prize of the Hungarian Government, 1958

Tibor Szele Prize of the Bolyai Mathematical Society, 1971

State Prize, Hungary, 1983

Wolf Prize, Israel, 1984 (shared with differential geometer Shiing-shen Chern, award $50,000 each)

Gold Medal of the Hungarian Academy of Science 1991

Honorary degrees

University of Wisconsin–Madison, U. S. A. 1973
Technische Universität Hannover, Germany 1977
University of York, U.K. 1979
University of Waterloo, Canada 1981
University of Western Ontario, London Ont., Canada 1985
Université de Limoges, France 1986
Cambridge University, U. K. 1991
Technion, Haifa, Israel 1992
Western Michigan University, Kalamazoo MI, U. S. A. 1992
Charles University, Prague, Czechoslovakia 1992
Eötvös University, Budapest, Hungary 1993
University of Illinois, Urbana-Champaign, U. S. A. 1993
University of Haifa, Israel 1994
University of Póznań, Poland 1994
Emory University, Atlanta, U. S. A. 1995

Elected to Member in Scientific Academies

1956 Hungarian Academy of Science (Corresponding Member)
1962 Hungarian Academy of Science (Ordinary Member)
1974 American Academy of Arts and Sciences (Foreign Honorary Member)
1977 Royal Netherlands Academy of Arts and Sciences[66]
1979 U. S. National Academy of Science
1985 Australian Academy of Science (Corresponding Member)
1988 Indian National Science Academy (Honorary Fellow)
1989 Royal Society (U. K.)
1994 Polish Academy of Science

Coauthors

The list below contains the names of 26 coauthors of Erdős, those who published at least ten joint papers with Erdős by the end of 1990.

The list is intended for the readers' amusement. We warn against any conclusions regarding the relative merits of these collaborations; for instance,

[66]This is the official translation of *Koninklijke Nederlandse Akademie van Wetenschappen.*

Kakutani with only 7 joint papers missed the list. It seems safe to say, however, that those on the list have had a prolonged and productive collaboration with Paul Erdős and his presence weighed (and in many cases continues to weigh) heavily in their mathematical careers.

The number of joint papers till 1990 (inclusive) precedes each name. Periods of active collaboration (by publication date) follow each name in [brackets]; * indicates that the number of joint papers continued to grow after 1990; ** indicates "rapid" growth. Deceased coauthors' dates of birth and death are in (parentheses).

50 Hajnal, András [1958 – *]
44 Sárközy, András [1966 – **]
32 Rényi, Alfréd (1921-70) [1949, 56–70]
31 Faudree, Ralph [1976 – *]
28 Turán, Pál (1910-76) [1934–41, 48–50, 55–73]
26 Schelp, Richard [1976 – **]
26 Szemerédi, Endre [1966 – 83]
24 Sós, Vera T. [1966, 70 – *]
23 Graham, Ronald L. [1972 – *]
23 Rousseau, Cecil [1976 – *]
21 Burr, Stephan [1975 – *]
20 Straus, E. G. (1921–83) [1953, 60–83]
18 Rado, Richard (1906–89) [1950 – 84]
17 Nathanson, Melvin [1975 – *]
17 Pomerance, Carl [1978 – **]
17 Simonovits, Miklós [1966 – *]
16 Spencer, Joel H. [1972 – **]
15 Milner, Eric [1966 – 78]
14 Nicolas, Jean-Louis [1975–82, 87– *]
12 Bollobás, Béla [1962, 73–80]
12 Chung, Fan R. K. [1979 – 87]
12 Hall, R.R. [1973 – 80]
12 Piranian, George [1967–88]
12 Selfridge, John [1967 – 88]
11 Reddy, A. L. [1973 – 78]
10 Pach, János [1980 – **]

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Sarah Rees, Miklós Sántha, Lex Schrijver, Jeff Shallit, R. K. Shyamasundar, Miklós Simonovits, Vera Sós, János Surányi, Robert Szelepcsényi, Éva Szilléry, Sundar Vishwanathan, and other friends around the globe for their advice and help in gathering and verifying data.

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Naturally I bear the responsibility for the many remaining errors and shortcomings. I will be grateful for any corrections.

I should like to express my gratitude to the Bolyai Society and especially to General Secretary Gyula O. H. Katona and Executive Director Cecilia Kulcsár for their infinite patience as well as their unrelenting prodding that made me, belatedly, begin, and eventually, stop writing this paper (necessarily unfinished; this paper is impossible to "complete"). My apologies are due to all those who contributed to this volume or otherwise have been waiting anxiously for it to come out, for having held up the publication for all too long. Paul Erdős's global 80th birthday party, which started in Kalamazoo[67] in June 1992, may perhaps end with this article.

While the world was celebrating, *Pali bácsi* completed about 120 new papers ...

May he continue to reveal his glimpses from the *Book*, for many years to come.

[67] *The Seventh Quadrennial International Conference on Graph Theory, Combinatorics, Algorithms, and Applications*, June 1-5, 1992, at Western Michigan University, was, as far as I know, the first meeting dedicated to Erdős's 80th birthday.

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