Feasible Commitment in Predicate Circumscription

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Abstract

In this paper we introduce a new semantic definition for predicate circumscription. We argue that the new definition is the most general one possible that captures the intuition, i.e., minimizing the objects that satisfy a given property. Yet, we can prove that it does not lead to inconsistency: If the original theory is consistent, then the circumscribed theory is also consistent. We also investigate quasiminimality as a midpoint between abstract minimality and the new definition to provide further comparison. We provide examples of consistent theories that have inconsistent set of consequences under the original and abstract definitions of circumscription. These theories have consistent consequences under the feasible commitment predicate circumscription. Both the feasible commitment predicate circumscription and quasiminimality coincide with circumscription based on abstract minimality whenever it produces sensible results. Therefore, these new definitions also coincide with the original definition of circumscription for well-founded theories.

1 Introduction

Circumscription is proposed by McCarthy [6] as a possible solution for the qualification problem in AI. He later introduced predicate circumscription[7] as a generalization of previous version which he called domain circumscription. Given a theory T and a set of predicates $\mathbf{P}$, predicate circumscription minimizes the extensions of $\mathbf{P}$ that satisfy T. The semantic counterpart to circumscriptive inference is defined to include only the models where the extensions of predicates in $\mathbf{P}$ is minimal. However, if the theory T does not have minimal models, then the circumscription of T results in inconsistency.

In the rest of the paper, we will use the word circumscription alone to mean predicate circumscription.

1.1 Preliminaries

In this subsection we introduce definitions and notations used in the rest of the paper. Our focus of attention is the variable predicate circumscription of a single predicate. It can easily be generalized to parallel predicate circumscription.

Definition 1.1 *Let $\mathcal{L}$ be a language for first order logic containing a predicate symbol* P *and predicates* $\mathbf{Z} = \{Z_1, \cdots, Z_n\}$*, and let* T *be an* $\mathcal{L}$*-theory. Let* $\mathfrak{A}$ *and* $\mathfrak{B}$ *be models of* T*. We say* $\mathfrak{B}$ *is a* $(\mathsf{P}; \mathbf{Z})$-predecessor *of* $\mathfrak{A}$*, and write* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$ *if*

1. *$\mathfrak{A}$ and $\mathfrak{B}$ have the same universe,*
2. *for all function and constant symbols $\mathsf{f} \in \mathcal{L}$, $\mathsf{f}^{\mathfrak{A}} = \mathsf{f}^{\mathfrak{B}}$,*
3. *for all relation symbols $\mathsf{R} \in \mathcal{L} - (\{\mathsf{P}\} \cup \mathbf{Z})$, $\mathsf{R}^{\mathfrak{A}} = \mathsf{R}^{\mathfrak{B}}$, and*
4. *$\mathsf{P}^{\mathfrak{A}} \subseteq \mathsf{P}^{\mathfrak{B}}$.*

A model $\mathfrak{A}$ of a theory T is $(\mathsf{P}; \mathbf{Z})$-*minimal* if $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$ implies $\mathfrak{A} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{B}$ for any other model $\mathfrak{B}$ of T (We will drop $(\mathsf{P}; \mathbf{Z})$ from the notation when it is clear from the context, and we will drop Z in the examples with no varying predicates.) The original semantics of circumscription takes into account all formulas that are true in the minimal models of a theory.

Definition 1.2 [2] *Let $\mathfrak{A}$ and $\mathfrak{B}$ be first-order structures.*

(a) A map $\pi : A \to B$ is called an isomorphism of $\mathfrak{A}$ onto $\mathfrak{B}$ *(written: $\pi : \mathfrak{A} \cong \mathfrak{B}$) iff*

1. *π is a bijection of A onto B ;*
2. *for n-ary relation symbol R and $a_0, \cdots, a_{n-1} \in A$,*
$$R^{\mathfrak{A}} a_0 \cdots a_n \text{ iff } R^{\mathfrak{B}} \pi(a_0) \cdots \pi(a_{n-1});$$
3. *for n-ary function symbol f and $a_0, \cdots, a_{n-1} \in A$,*
$$\pi(f^{\mathfrak{A}}(a_0, \cdots, a_{n-1})) = f^{\mathfrak{B}}(\pi(a_0) \cdots \pi(a_{n-1}));$$
4. *for every constant symbol c, $\pi(c^{\mathfrak{A}}) = c^{\mathfrak{B}}$.*

(b) $\mathfrak{A}$ and $\mathfrak{B}$ are said to be isomorphic (written: $\mathfrak{A} \cong \mathfrak{B}$) if there is an isomorphism $\pi : \mathfrak{A} \cong \mathfrak{B}$.

1.2 The Inconsistency Problem

A problem that effects all previously considered definitions of circumscription is that satisfiable theories may have inconsistent consequences. One approach to this problem is to restrict the application of circumscription to a class of theories where it never leads to a contradiction. For instance, the original definition does not lead to inconsistency when restricted to well-founded theories. Since our discussion is restricted to single predicate circumscription, we give the two definitions below for a single predicate whereas they are originally for multiple predicates.

Definition 1.3 (Lifschitz [5]) *A theory* T *is* well-founded w.r.t $(\mathsf{P};\mathbf{Z})$ *if for each model* $\mathfrak{A}$ *of* T*, there exists a* $(\mathsf{P};\mathbf{Z})$*-minimal model* $\mathfrak{B}$ *such that* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$.

However, there are perfectly reasonable theories, such as the simple theory of natural successor (the example given in Etherington [3]), that are not well-founded.

Another approach to the same problem is to weaken circumscription by generalizing the minimality criterion which will accordingly increase the class of theories that can be consistently handled.

Definition 1.4 (Liau et. al. [4]) *Given a theory* T*, a model* $\mathfrak{A}$ *of* T *is* abstractly $(\mathsf{P};\mathbf{Z})$-minimal *if for each model* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$, $\mathfrak{B} \cong \mathfrak{A}$.

Since a theory can have abstractly minimal models without having minimal models (i.e., theory of successor), circumscription via abstract minimality is more general than McCarthy's original definition.

One of our proposals is a further generalization of minimality criterion by the introduction of quasiminimal models. A model is quasiminimal w.r.t predicate P and a set of varying predicates $\mathbf{Z}$ if any of its $(\mathsf{P};\mathbf{Z})$-predecessors has a predecessor isomorphic to the model. Abstractly minimal models are also quasiminimal. Hence, by quasiminimality, we enlarge the set of predicates suitable for circumscription.

Although generalizing the minimality criterion will take us to a larger class of theories, it is possible to find unsuitable theories for both abstract minimality and quasiminimality. We give an example where a theory can have abstractly minimal models w.r.t. $(\mathsf{P};\mathbf{Z})$, yet there are also models of the same theory without abstractly minimal predecessors. This observation shows us that the circumscription based on generalized minimal models can still be too strong. Of course, one can restrict the applicability of these weak circumscriptions to classes of theories where the corresponding "well-foundedness" condition holds. However, we will take a different approach.

Our work is motivated by the idea that we can give a concise semantic definition for predicate circumscription that includes all theories with a consistent set of consequences.

2 Feasible Commitment Predicate Circumscription

The original definition of circumscription is based on minimal models. However, if a theory has no minimal models, then circumscribing it gives an inconsistent theory. Davis [1] and Etherington [3] provide examples of such theories for domain and predicate circumscription, respectively. Liau et. al. [4] presents Etherington's example and shows how their definition of abstract circumscription solves the inconsistency problem for the given example. Our examples include theories which do not even have abstractly minimal models. Yet, there are intuitive circumscriptive consequences of these theories.

2.1 Motivating Examples

Example 1 *The theory of dense linear order without end points consists of the following axioms,* $\mathrm{T}_{\mathrm{DLOw/oEP}}$*:*

1. $\forall x \ \ (\neg x < x)$,
2. $\forall x \forall y ((x < y) \vee (x = y) \vee (y < x))$,
3. $\forall x \forall y \forall z ((x < y) \wedge (y < z) \rightarrow (x < z))$,
4. $\forall x \forall y ((x < y) \rightarrow \exists z (x < z) \vee (z < y))$,
5. $\forall x \exists y \ \ y < x$,
6. $\forall x \exists y \ \ x < y$.

$\mathrm{T}_1 = \mathrm{T}_{\mathrm{DLOw/oEP}} \cup \{\forall x (P(x) \wedge y < x \rightarrow P(y)), \exists (x > 0) P(x)\}$.

T_1 includes an additional predicate P whose extension is downward closed. T_1 is the theory of the Dedekind cut of an infinitesimal. T_1 does not have P-minimal models. For any model of T_1 we can get to another model with a smaller extension of P by cutting off an end-segment from the sequence of elements in the previous extension. Moreover, T_1 does not have abstractly minimal models either. Because, in a model of T_1 the extension of P may or may not contain a maximal element. Let $\mathfrak{A} = \langle \mathrm{Q}, <, 0, \mathsf{P}^{\mathfrak{A}} \rangle$ be a model of T_1, where $\mathsf{P}^{\mathfrak{A}} = \{x : x \leq r, r \geq 0\}$. We can get to a model $\mathfrak{B} = \langle \mathrm{Q}, <, 0, \mathsf{P}^{\mathfrak{B}} \rangle$, where $\mathsf{P}^{\mathfrak{B}} = \{x : x < s, s \geq 0\}$, $s < r$, by removing the final elements from $\mathsf{P}^{\mathfrak{A}}.\mathfrak{B}$ is still a model of T_1, however, $\mathsf{P}^{\mathfrak{B}}$ does not have a maximal element. Similarly, if $\mathsf{P}^{\mathfrak{A}} = \{x : x < r\}$, we can obtain $\mathsf{P}^{\mathfrak{B}}$ with a maximal element. Thus, there is always a $\mathfrak{B} \preceq_{\mathsf{P}}^{\mathrm{T}_1} \mathfrak{A}$ which is not isomorphic to $\mathfrak{A}$.

Intuitively, however, T_1 should minimally entail those formulas that become true and stay true in the models of T_1 that are pre-ordered by $(\mathsf{P}; \mathbf{Z})$-predecessor relation. For example, in the models of T_1, $\neg P(1)$ becomes true when a finite end segment is cut off, and stays true afterwards.

Example 2 $\mathrm{T}_2 = \mathrm{PA} \cup \{\exists x \;\; big(x)\} \cup \{\forall x \forall y (x < y \wedge big(x) \rightarrow big(y))\}$.

The set PA is the set of axioms of Peano Arithmetic. For any model $\mathfrak{A}$ of T_2, there exists a predecessor $\mathfrak{B}$, $\mathfrak{B} \models \mathrm{T}_2$, but not isomorphic to $\mathfrak{A}$ due to the axioms in PA. Indeed, there are infinitely many nonisomorphic models of T_2. However, as in the previous example, there are formulas that eventually become true in the models of T_2 as the extension of big gets smaller. As an example, let's take $\neg\mathsf{big}(\overline{17})$.

Let $\mathfrak{A} = \langle \omega, 0, s, +, ., <, \omega \rangle$ (s is the successor function) where $\mathsf{big}^{\mathfrak{A}} = \omega$. $\mathfrak{A} \not\models \neg\mathsf{big}(\overline{17})$. But, there exists $\mathfrak{B} = \langle \omega, 0, s, +, ., <, \{x : x > 18\} \rangle$ and $\mathfrak{B} \models \neg\mathsf{big}(\overline{17})$. In addition, for all models $\mathfrak{C}$ that are big-predecessors of $\mathfrak{B}$, $\mathfrak{C} \models \neg\mathsf{big}(\overline{17})$. However, none of those predecessors are isomorphic to $\mathfrak{B}$. Thus, T_2 has no minimal or abstractly minimal models. Yet, by intuition, T_2 should minimally entail $\neg\mathsf{big}(\overline{17})$. We can say that $\neg\mathsf{big}(\overline{17})$ becomes true in "perpetuity". Feasible commitment semantics for circumscription exactly captures this point.

Example 3 $\mathrm{T}_{\mathrm{DLO}} = \mathrm{T}_{\mathrm{DLOw/oEP}} \setminus \{\forall x \exists y \;\; y < x, \forall x \exists y \;\; x < y\}$,
$\mathrm{T}_3 = \mathrm{T}_{\mathrm{DLO}} \cup \{\exists x \;\; small(x), \forall x \forall y (small(y) \wedge x < y \rightarrow small(x))\}$.

Some of the models of T_3 has a domain with a lower bound. Therefore, T_3 has small-minimal models where the extension of small is a singleton set containing the lower bound. However, some models of T_3 does not have a lower bound, consequently, these models do not have small-minimal predecessors. Any definition of circumscription which takes into account the $(\mathsf{P}; \mathbf{Z})$-minimal models only is too strong. For instance, for all small-minimal models $\mathfrak{A}$ of T_3, $\mathfrak{A} \models \exists x \forall y \;\; x \leq y$. In this case, abstract circumscription is too strong too since any $(\mathsf{P};\mathbf{Z})$-minimal model is also abstractly $(\mathsf{P};\mathbf{Z})$-minimal; yet, the models with no predecesors are not abstractly minimal either.

2.2 A New Semantic Definition for Predicate Circumscription

Our primary motivation is to give the most general semantics for the predicate circumscription formalism that captures the intuition. The examples in the previous section provides us what this new semantic definition should accout for. In addition, such a definition should not lead to any inconsistencies. We will prove later that if the original theory is consistent, the circumscribed theory is also consistent.

The models of a theory T are pre-ordered w.r.t the relation $(\mathsf{P};\mathbf{Z})$-predecessor. For any model $\mathfrak{B}$ of T, if a formula φ holds in $\mathfrak{B}$ and in all of its $(\mathsf{P};\mathbf{Z})$-predecessors, we say $\mathfrak{B}$ is committed to φ. A model $\mathfrak{A}$ of T has a feasible commitment to φ if $\mathfrak{A}$ has a $(\mathsf{P};\mathbf{Z})$-predecessor that is committed to φ. A formula φ is a feasible commitment circumscriptive consequence of T if every model of T has a feasible commitment to φ. We define the commitment of a model as a forcing relation.

Definition 2.1 *Let* T *be a theory where* P *is a predicate and* $\mathbf{Z}$ *is a set of predicates in* T.

1. *For* $\mathfrak{A} \models \mathrm{T}$*,* $\mathfrak{A}$ is committed to φ *(notation:* $\mathrm{T}, \mathfrak{A} \Vdash^{\mathsf{P};\mathbf{Z}} \varphi$*) iff for all* $\mathfrak{B} \models \mathrm{T}$ *whenever* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$*, then* $\mathfrak{B} \models \varphi$*.*
2. φ is a feasible commitment circumscriptive (FCC) consequence of T *(notation:* $\mathrm{T} \models^{\mathsf{P};\mathbf{Z}}_{FC} \varphi$ *) iff for all* $\mathfrak{A} \models \mathrm{T}$ *there exists* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$ *such that* $\mathfrak{B}$ *is committed to* φ*,* $\mathrm{T}, \mathfrak{B} \Vdash^{\mathsf{P};\mathbf{Z}} \varphi$*.*
3. $CLOSURE(\mathrm{T}) = \{\varphi : \mathrm{T} \models^{\mathsf{P};\mathbf{Z}}_{FC} \varphi\}$.

In the following examples of predicate circumscription, there are no varying predicates, therefore we will drop $\mathbf{Z}$ from the notation.

Example 1 *(reconsidered)*

The sentence $\neg P(1)$ is in $CLOSURE(\mathrm{T}_1)$. To prove $\mathrm{T}_1 \models^{\mathsf{P}}_{FC} \neg P(1)$, we observe that any model $\mathfrak{A} = \langle \mathrm{Q}, <, 0, \mathsf{P}^{\mathfrak{A}} \rangle$ of theory T_1 has either

- $\mathsf{P}^{\mathfrak{A}} = \{x : x < r\}$ where $r < 1$ $(\mathrm{T}, \mathfrak{A} \Vdash^{\mathsf{P}} \neg P(1))$, or

- $\mathfrak{B} = \langle \mathrm{Q}, <, 0, \{x : x < 1\}\rangle$ as a P-predecessor $(\mathrm{T}, \mathfrak{B} \Vdash^{\mathsf{P}} \neg P(1))$.

In both cases, $\mathrm{T}_1 \models^{\mathsf{P}}_{FC} \neg P(1)$.

Example 2 *(reconsidered)*

For any n we can name, $\mathrm{T}_2 \models^{\mathsf{big}}_{FC} \neg big(\overline{n})$. The following is a more interesting FCC consequence of T_2 about the asymptotic behaviours of two functions:
$\mathrm{T}_2 \models^{\mathsf{big}}_{FC} \forall x(big(x) \rightarrow x^2 + \overline{100} < x^2 + x)$.

Example 3 *(reconsidered)*

FCC circumscription considers all the models including the ones with no lower bound, not only abstractly minimal ones with a lower bound. Therefore,

$$\mathrm{T}_3 \not\models^{\mathsf{small}}_{FC} \exists x \forall y \quad x \leq y.$$

The following is the proof that feasible commitment predicate circumscription always results in satisfiable theories if the original theory is consistent.

Lemma 2.2 *If* $\mathrm{T}, \mathfrak{A} \Vdash^{\mathsf{P};\mathsf{Z}} \varphi$*, then* $\mathfrak{A} \models \varphi$.

Proof: Since $\mathfrak{A} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$ for all $\mathfrak{A}$, it implies $\mathfrak{A} \models \varphi$ by definition. $\square$

Theorem 2.3 *If* $\mathrm{T} \not\vdash \perp$ *then* $\mathrm{T} \not\models^{\mathsf{P};\mathsf{Z}}_{FC} \perp$.

Proof: Suppose T is consistent, i.e. , $\mathrm{T} \not\vdash \perp$. By the completeness theorem, there exists a model $\mathfrak{A}$ that satifies T. Suppose $\mathrm{T} \models^{\mathsf{P};\mathsf{Z}}_{FC} \perp$, then, by definition 2.1, there exits a model $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$ such that $\mathrm{T}, \mathfrak{B} \Vdash^{\mathsf{P};\mathsf{Z}} \perp$. However, by the lemma 2.2, $\mathfrak{B} \models \perp$. No model can have $\mathfrak{B} \models \perp$, contradiction. $\square$

3 Quasiminimality and Predicate Circumscription

In order to solve the inconsistency problem, we can also generalize the minimality criterion on which the semantics of circumscription is based. Previously proposed sematics for predicate circumscription [7] [4] are based on a minimality criterion on the models of a given theory. We get unsatisfiable theories resulting from circumscription when the given theory does not have any models satisfying the minimality criterion.

We would like to generalize minimality criterion further to include a larger class of theories. Quasiminimality is a further step in this direction. Abstract minimality requires that all predecessors of a model be isomorphic to it for that model to be abstractly minimal. Abstract minimality is an interesting concept. It allows us to consider structurally equivalent models as one equivalence class. However, a model may not have all isomorphic predecessors. Yet, it may have isomorphic predecessors often enough so that all of its predecessors lead us to an isomorphic one. For instance, T_1 in example 1 has three different models up to isomorphism: the Dedekind cut of a rational with and without an upper bound (models $\mathfrak{A}$ and $\mathfrak{B}$ in the example), and that of an irrational. Although each model has a nonisomorphic predecessor, we can find an isomorphic model among the predecessors of every nonisomorphic predecessor by further removing elements from the end.

Definition 3.1 *Let* T *be a theory,* P *is a predicate and* $\mathbf{Z}$ *is a tuple of predicates in* T. $\mathfrak{A}$ *is* a quasi-minimal model of T with respect to P and $\mathbf{Z}$ *iff for all models* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$ *there exists* $\mathfrak{C} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$ *such that* $\mathfrak{C} \cong \mathfrak{A}$.

In the following discussion, the term "quasiminimal" refers to quasiminimality with respect to a circumscribed predicate P and a tuple of varying predicates $\mathbf{Z}$.

Theorem 3.2 *Let* $\mathfrak{A}$ *be a quasiminimal model of* T*, and* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$*, then* $\mathfrak{B}$ *is also quasiminimal.*

Proof: Suppose $\mathfrak{B}'$ is any $(\mathsf{P};\mathbf{Z})$-predecessor of $\mathfrak{B}$ ($\mathfrak{B}' \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$). We will show that there exists another model $\mathfrak{B}'' \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}'$ such that $\mathfrak{B}'' \cong \mathfrak{B}$.

Since $\mathfrak{B}' \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$, and $\mathfrak{A}$ is quasiminimal, by definition 3.1, there exists $\mathfrak{A}' \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}'$ such that $\mathfrak{A}' \cong \mathfrak{A}$. Let ϕ be isomorphism between $\mathfrak{A}'$ and $\mathfrak{A}$. It suffices if can construct $\mathfrak{B}''$ by giving an interpretation of predicate P. We define $\mathfrak{B}''$ as follows.

Let $a_0, \cdots, a_{n-1}$ be elements in the domain of $\mathfrak{A}$. Choose $\mathsf{P}^{\mathfrak{B}''}$ such that

$$\mathsf{P}^{\mathfrak{B}''} a_0, \cdots, a_{n-1} \text{ iff } \mathsf{P}^{\mathfrak{B}} \phi^{-1}(a_0), \cdots, \phi^{-1}(a_{n-1})$$

(i.e., $\mathsf{P}^{\mathfrak{B}''} = \phi(\mathsf{P}^{\mathfrak{B}})$). Since $\mathsf{P}^{\mathfrak{B}} \subseteq \mathsf{P}^{\mathfrak{A}}$ implies $\phi(\mathsf{P}^{\mathfrak{B}}) \subseteq \phi(\mathsf{P}^{\mathfrak{A}})$, and
$\phi(\mathsf{P}^{\mathfrak{A}}) = \mathsf{P}^{\mathfrak{A}'} \subseteq \mathsf{P}^{\mathfrak{B}'}$,
we have

$$\mathsf{P}^{\mathfrak{B}''} = \phi(\mathsf{P}^{\mathfrak{B}}) \subseteq \phi(\mathsf{P}^{\mathfrak{A}}) = \mathsf{P}^{\mathfrak{A}'} \subseteq \mathsf{P}^{\mathfrak{B}'}.$$

Thus, $\mathfrak{B}'' \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}'$. $\square$

3.1 Comparison of the New Semantic Definition and Quasiminimality

The generalization of minimality criterion alone is not enough to avoid inconsistent theories. In example 2, T_2 do not have any isomorphic models. Therefore, T_2 does not have any quasiminimal models. We restrict the applicability of circumscription based on quasiminimal models to supported theories.

Definition 3.3 *A theory* T *is supported by quasiminimal models if for all models* $\mathfrak{A}$ *of* T *there exists a quasiminimal model* $\mathfrak{B}$ *of* T *w.r.t.* $(\mathsf{P};\mathsf{Z})$ *and* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$*.*

T_2 is not a supported theory according to the above definition. However, if a theory is supported, then its FCC consequences coincide with the ones that hold in its quasiminimal models. In what follows, we will drop P and Z from the notation when it is clear from the context.

Theorem 3.4 *Let* T *be a supported theory. If* φ *holds in all quasiminimal minimal models of* T *w.r.t.* $(\mathsf{P};\mathsf{Z})$*, then* $\mathrm{T} \models^{\mathsf{P};\mathsf{Z}}_{FC} \varphi$*.*

Proof: Let φ be a formula that is true in all quasiminimal models of T. Let $\mathfrak{A}$ be a model of T . Since T is supported, there exits a quasiminimal model $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$. By the assumption, $\mathfrak{B} \models \varphi$. By theorem 3.2, whenever $\mathfrak{C} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$, then $\mathfrak{C}$ is quasiminimal, too. By the assumption again, $\mathfrak{C} \models \varphi$. Hence, for all $\mathfrak{C} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$, $\mathfrak{C} \models \varphi$. Thus, by definition 2.1, $\mathrm{T}, \mathfrak{B} \Vdash \varphi$. Therefore, $\mathrm{T} \models^{\mathsf{P};\mathsf{Z}}_{FC} \varphi$. $\square$

Theorem 3.5 *For any first order formula* φ*, if* $\mathrm{T} \models^{\mathsf{P};\mathsf{Z}}_{FC} \varphi$*, then* φ *holds in all quasiminimal models of* T*.*

Proof: Let $\mathfrak{A}$ be a quasiminimal model of T. Since $\mathrm{T} \models^{\mathsf{P};\mathsf{Z}}_{FC} \varphi$, there exists $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$ such that $\mathrm{T}, \mathfrak{B} \Vdash \varphi$. That is, for all $\mathfrak{C} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$, $\mathfrak{C} \models \varphi$. However, since $\mathfrak{A}$ is quasiminimal, for all such $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{A}$, there exists a model $\mathfrak{C} \preceq^{\mathrm{T}}_{\mathsf{P};\mathsf{Z}} \mathfrak{B}$, such that $\mathfrak{C} \cong \mathfrak{A}$. But, since $\mathrm{T}, \mathfrak{B} \Vdash \varphi$, all predecessors of $\mathfrak{B}$ satisfy φ, therefore $\mathfrak{C} \models \varphi$. By the isomorphism lemma, $\mathfrak{A} \models \varphi$. $\square$

3.2 Coincidence with Abstract Predicate Circumscription

Liau et al.[4] proves that the abstract predicate circumscription coincides with the original predicate circumscription for well-founded theories. We will show the coincidence of feasible commitment predicate circumscription with abstract predicate circumscription in case of theories with abstractly minimal models.

Definition 3.6 *A theory* T *is* supported by abstractly $(\mathsf{P};\mathbf{Z})$-minimal models *if for all models* $\mathfrak{A}$ *of* T*, there exists an abstractly minimal model* $\mathfrak{B}$ *of* T *such that* $\mathfrak{B} \preceq^{\mathrm{T}}_{\mathsf{P};\mathbf{Z}} \mathfrak{A}$.

Lemma 3.7 *If theory* T *is supported by abstractly minimal models, then its abstractly* $(\mathsf{P};\mathbf{Z})$*-minimal models are exactly the quasiminimal models.*

Theorem 3.8 *Let* T *be a theory supported by abstractly* $(\mathsf{P};\mathbf{Z})$*-minimal models. Then,* $\mathrm{T} \models^{\mathsf{P};\mathbf{Z}}_{FC} \varphi$ *iff* φ *holds in all abstractly minimal models of* T.

Proof: If part: Suppose φ holds in all abstractly minimal models of T. By the lemma 3.7, φ holds in all quasiminimal models of T. Then, Theorem 3.4 implies $\mathrm{T} \models^{\mathsf{P};\mathbf{Z}}_{FC} \varphi$.

Only-if part: $\mathrm{T} \models^{\mathsf{P};\mathbf{Z}}_{FC} \varphi$ implies that φ holds in all quasiminimal models, by Theorem 3.5. Since T is supported, by the lemma 3.7, all of it quasiminimal models are abstractly minimal. Therefore, φ holds in all abstractly minimal models of T. □

4 Conclusion

The semantics for predicate circumscription is based on minimal models. We have shown that circumscription defined by a minimality criterion can be too strong, in particular, may lead to inconsistency. Quasiminimality gives us a more general minimality criterion than abstract minimality does, however, circumscription based on quasiminimal models can still be too strong. Feasible commitment predicate circumscription defines the most general circumscription semantics that never leads to inconsistency.

4.1 Work in Progress

In this paper, we have only provided semantics of two new definitions for circumscription: feasible commitment predicate circumscription and predicate circumscription based on quasiminimality. Circumscription is generally defined as a syntactic transformation of a given theory to its circumscribed form. The circumscriptive inference is defined via the consequences of the circumscribed form of a theory. We instead took a different approach, we defined the model theoretic properties first. The notion of inference that is sound and complete is still an open problem. Our work is in progress.

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